Separation vortices and surface shapes

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Separation in fluid flows is a complex phenomenon, which can give rise to interesting structures in free surface flows or in surfaces between two fluids. Separation occurs in flows that are predominantly moving in one direction, and at separation part of the flow acquires velocities opposite to this direction. When separation develops, it typically occurs in a finite region of the flow, called a *separation vortex* - or *bubble*. The theory of separation is notoriously difficult. The reason is that separation occurs in connection with boundary layers at moderate Reynolds number, and the standard boundary layer theory due to Prandtl becomes singular at separation points.

A simple example of separation is found in the circular hydraulic jump. As shown in the sketch on Figure 1, a separation vortex forms near the bottom just outside the jump, and this is what makes the external flow thicker.

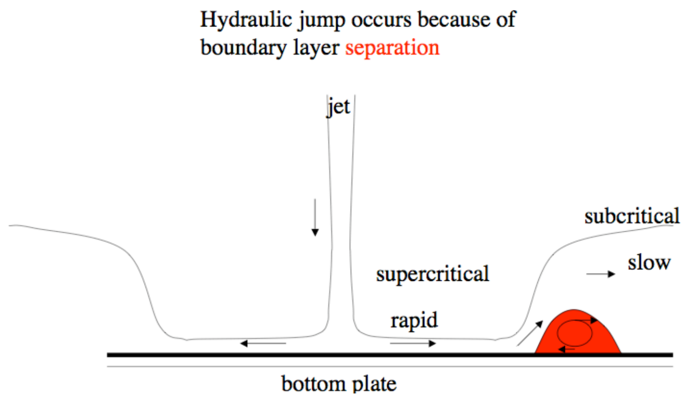

Figure 1. Sketch of the flow through a section of a circular hydraulic jump. The separation vortex is clearly shown on the bottom right.

This flow structure can be captured by averaging techniques, as shown in Figure 2, by modeling the velocity profile as a cubic polynomial [1].

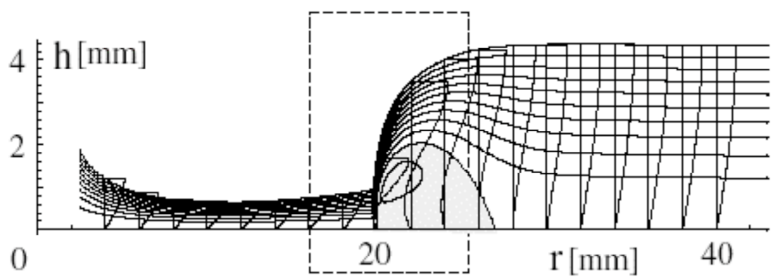

Figure 2. Height profile and flow structure for a circular hydraulic jump computed by averaged boundary layer theory through the separation point. From [1].

Another situation in which separation - in this case time dependent - plays a major role for pattern formation, is the generation of sand-ripples under oscillatory flow. In experiments (Figure 3), a tray of sand is moved back and forth in an aquarium and the ripple structure is generated by the separation vortices that form on the lee side of each vortex in each stroke. Here, the problem is complicated by the fact that the vortex generation is amplified by the time dependence.

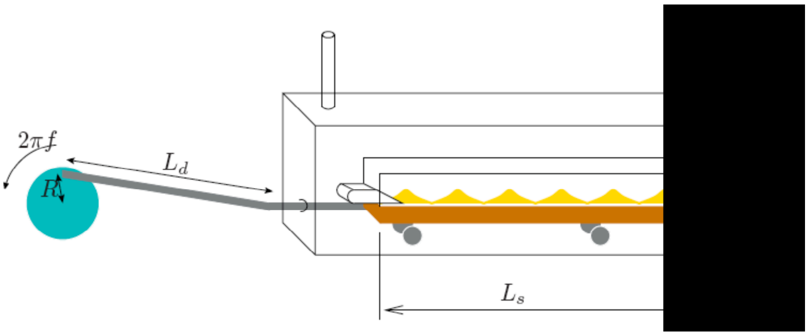

Figure 4. Vortex ripples forming on an oscillating tray of sand in water. From [2]

We have recently formulated an amplitude equation, which can model this type of pattern formation [2]. In terms of the local ripple height h and its average $\bar{h}$ in a 1-d system the equation can be written

$$h_t = -\epsilon(h - \bar{h}) + A\big((h_x)^2 - \theta^2\big)h_{xx} - \nu h_{xxxx} + \delta\big((h_x)^2\big)_{xx}$$

and without actually solving the time dependent separation problem on an arbitrary surface, it does allow us to describe both the creation of the ripples and the interrupted coarsening process leading to their final shape.

These systems have very interesting secondary instabilities, which come from secondary bifurcations of the separation vortices. In the circular hydraulic jump, increase of the height of the outside layer can lead to a new flow structure with an additional separation vortex ("roller") at the surface (Figure 4 (left)) and this often leads to a state with spontaneously broken axial symmetry (Figure 4 (right)).

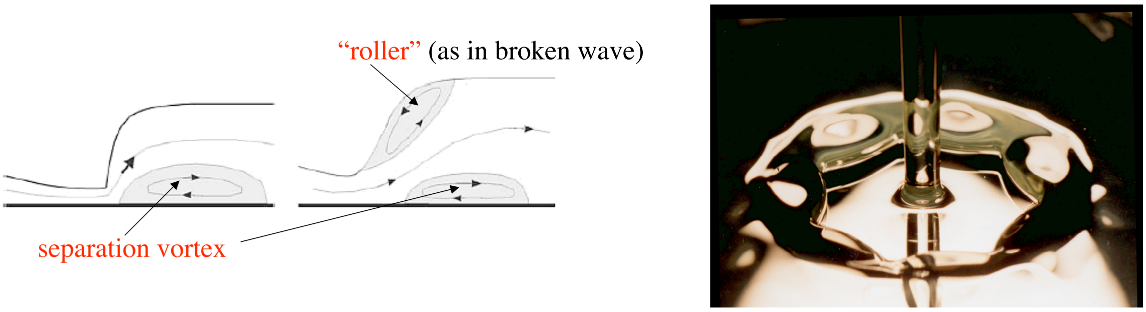

Figure 5. "Wave breaking" transition for a circular hydraulic jump. Increasing the height of the outer layer leads to a new flow structure with a separation vortex ("roller") at the surface. This roller spontaneously breaks the axial symmetry and the surface shape becomes a "polygon"[3].

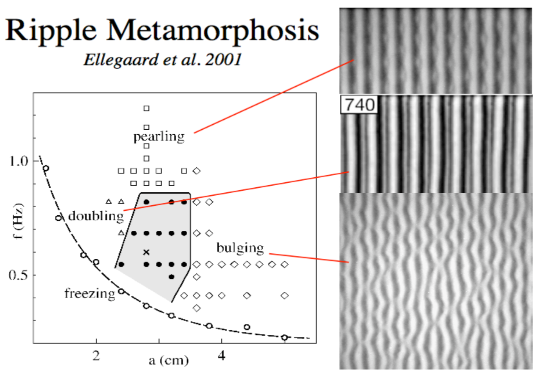

Figure 6. Instabilities of ripple patterns (from [4]).

In the case of sand-ripples, a change in amplitude or frequency of the oscillation leads to a sequence of new patterns as shown in Figure 5. The middle one ("doubling") can be captured by the above amplitude equation, but the two others remain to be understood.

References:
1. T. Bohr et al. *Phys. Rev. Lett.* **79** 1038 (1997)
2. T. Schnipper et al., preprint (2008).
3. C. Ellegaard et al. *Nature* **392**, 767(1998)
4. J. L. Hansen et al. *Nature* **410**, 324 (2001)