Chaotic scattering of two identical point vortex pairs

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Elementary interactions of point vortices are of interest in their own right and to the modeling of 2D turbulent flows. The Hamiltonian dynamics of N point vortices has been extensively studied. In the unbounded plane. The system is known to be integrable for $N \leq 3$, [1]. A pair of vortices of opposite circulations propagates along a straight line. Two vortex pairs may be brought to collide, constituting a proper scattering process. The scattering outcome depends on three parameters: The angle of incidence, the (kinetic) energy difference between the two pairs, and an impact parameter related to the distance of closest approach of the pair trajectories in the absence of interaction. Furthermore, the problem is non-integrable with chaotic regimes, so it is considerably more complicated than the scattering problems traditionally studied in particle mechanics. Vortex scattering has been studied in [2, 3, 4, 5]. We may summarize the findings in these papers by saying that that chaotic scattering regimes occur when the two pairs have constituent vortices of different circulations or are of very different energies.

I provide numerical evidence that chaotic scattering occurs even in the collision of identical vortex pairs both as regards circulations and energies (i.e., initial vortex separations), see [6]. The mechanisms leading to chaotic scattering are different from the "slingshot effect" identified by Price [4] and occur in a different region of the four-vortex phase space.

I will present analytical results based on the known integrals of motion, i.e., energy, linear and angular momenta (in this case the same as linear and angular impulse). These lead to constraints on the scattering outcome, but do not provide a solution to the problem, except in certain special integrable cases.

In the chaotic regimes, the duration of the scattering process plotted as a function of impact parameter has sharp peaks, where the interaction time diverges. Vortex trajectories in a chaotic scattering process are shown in figure 1. The duration and the outcome depend sensitively on initial conditions. Scattering diagrams corresponding to these regimes display a self-similar fractal structure, which is the hallmark of chaos. The mechanism of chaotic scattering is that the system approaches an unstable periodic motion and orbits close to it for a time, before breakup occurs and two new vortex pairs are emitted from the scattering region. In some cases the periodic motion corresponds to well known motions of an integrable system. We have identified several such cases.

An interesting feature of the scattering is that energy may be transferred from one pair to another, providing an interesting perspective for statistical analysis of turbulent 2D flows.

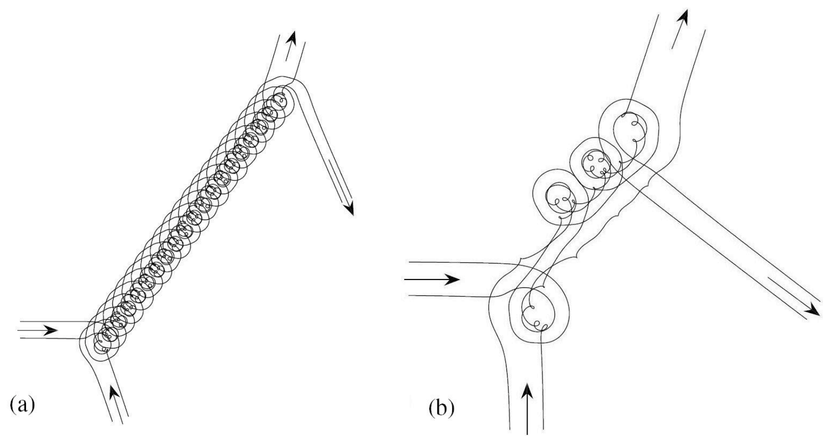

Figure 1: Chaotic scattering of vortex pairs. (a): Oblique angle of incidence. (b) right angle of incidence. The system approaches a periodic state, which is unstable, leading to eventual breakup and emission of the outgoing vortex pairs.

The interaction duration is long, leading to sensitive dependence on initial conditions. Trajectories are obtained by numerical simulation. Note that while the incoming pairs are identical in both collisions, the outgoing vortex pairs in both cases have different sizes, corresponding to different energies.

References

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