On the relationship between a vortex and vorticity

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Though the intuitive idea of a vortex is of fundamental importance for fluid mechanics, there is still no consensus on the generally acceptable and rigorous definition of this distinct flow phenomenon. Nevertheless, fluid vortices have always been somehow related to a quite mathematically rigorous and physically well-established quantity: vorticity. This quantity, expressing an average angular velocity of fluid elements, attains high values in vortex cores (relative to its environment), however, this quantity cannot distinguish between shearing motions and the actual swirling motion of a vortex and misrepresents vortex geometry.

A large number of vortex-identification methods, vortex definitions, and vortex-core visualization techniques have been proposed (see e.g. Jeong and Hussain 1995, Kida and Miura 1998, Cucitore et al. 1999, Roth 2000, Chakraborty et al. 2005, Kolář 2007, and the references therein). The recent method of Kolář (2007) offers a certain qualitative "comeback" of vorticity to vortex identification, namely its specific portion labelled *residual* vorticity which is obtained after the extraction of a pure shearing motion and represents a direct measure of the swirling motion. The shearing motion itself is responsible for a specific portion of vorticity labelled *shear* vorticity. In 2D, there is a straightforward interpretation of the *residual* vorticity in terms of the least-absolute-value angular velocity of all line segments, within the flow plane, going through the given point, Fig.1.

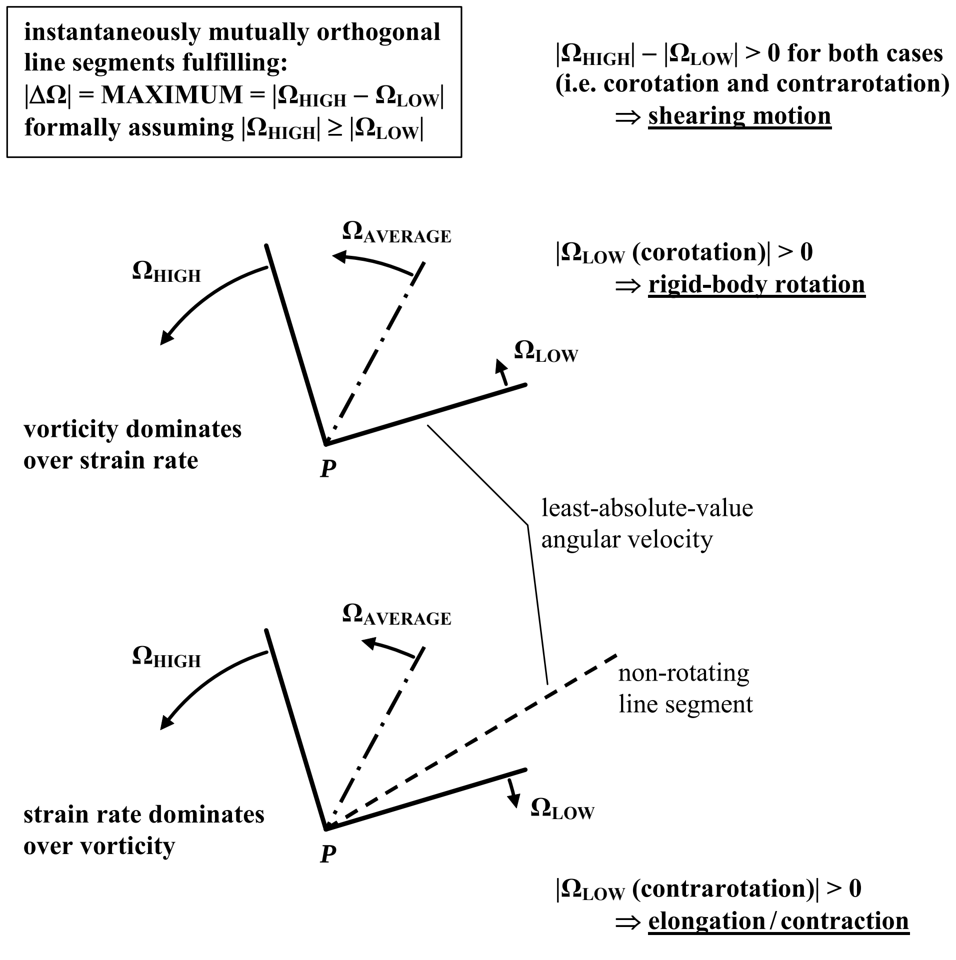

Fig. 1 Interpretation of the *residual* vorticity in 2D: the least-absolute-value angular velocity.

The given vortex-identification method employs an additive vorticity decomposition which is an outcome of the triple decomposition of the relative motion near a point (TDM), proposed in Kolář (2007), where a closely related 3D algorithm for the triple decomposition of $\nabla \mathbf{u}$ is presented. TDM aims at the extraction of a so-called "effective" pure shearing motion. Accordingly $\nabla \mathbf{u} = \mathbf{S} + \mathbf{\Omega} = (\textit{residual tensor} = \mathbf{S}_{\mathrm{RES}} + \mathbf{\Omega}_{\mathrm{RES}}) + (\textit{shear tensor})$ where the residual tensor has to do with elongation through $\mathbf{S}_{\mathrm{RES}}$ and rigid-body rotation through $\mathbf{\Omega}_{\mathrm{RES}}$ while the shear tensor — of a defined purely asymmetric tensor structure — represents a pure shearing motion. The decomposition algorithm is based on the extraction of the shear tensor from $\nabla \mathbf{u}$ on condition that it is maximizing the effect of extraction. It is guaranteed by minimizing the absolute tensor value of the residual tensor or, equivalently, by maximizing the quantity $|S_{12}\Omega_{12}| + |S_{23}\Omega_{23}| + |S_{31}\Omega_{31}|$ for $\nabla \mathbf{u}$ viewed in the so-called *basic reference frame* (BRF). This frame is determined simultaneously on the basis of **S** and **Ω** unlike the system of principal axes of **S**, which is minimizing (to zero) the frame-dependent value of $|S_{12}\Omega_{12}| + |S_{23}\Omega_{23}| + |S_{31}\Omega_{31}|$. Consequently, these two reference frames are considered as "antipoles" for depicting $\nabla \mathbf{u}$.

The present conference paper deals in detail with various physical aspects of the above mentioned concept of the relationship between a vortex and vorticity and the associated local flow kinematics. The most important aspect of the given identification approach is that the definition of a vortex through the *residual* rigid-body rotation is tied together — through the TDM — with the definition of a pure shearing motion. The same holds for an irrotational straining as the three elementary parts of the TDM are mutually conditionally balanced. The flow geometry obtained in the BRF reveals that for frequently equal or nearly equal leading-diagonal elements the coordinate plane exhibiting elongation excludes rigid-body rotation in this plane and vice versa, hence the planes of these *residual* motions are perpendicular. A non-destructive nature of the superposition of the TDM parts is discussed. Some properties of the BRF and *residual* vorticity in 3D are investigated. An arbitrary non-zero deviatoric strain-rate tensor and an arbitrary non-zero vorticity tensor produce a non-zero shear tensor. Various shear-tensor structures are uniquely interpreted by means of the local kinematics of shearing elements — planes, lines, or points, depending on the flow complexity in 3D. An interesting, but controversial, idea of the vortex-identification requirement of allowance for an arbitrary axial strain (Wu et al. 2005) vs. the requirement of orbital compactness (Chakraborty et al. 2005) is examined using the TDM. It is found that while stretching (uniaxially or radially) the local vortical motion near a point, there is an inherent objective physical bound for the amount of stretching to identify the examined point as part of a vortex. This bound is just the local 3D pure shearing motion of material points introduced in the frame of the TDM. The qualitative model of the TDM (see Fig. 1 of Kolář 2007) may help to qualitatively distinguish between vortex sheets and tubes. The asymptotic behaviour of a local vortex-shear interaction is examined in view of the meaningful local intensity of a vortex. The natural requirement of vortex-axis uniqueness for each connected vortex region is commented using simple flow examples of vortex-vortex interaction. The concept of *residual* circulation (calculated as a surface quadrature of the *residual* vorticity) for the description of vortex strength is discussed. An attempt to compare qualitatively the TDM vorticity decomposition with other known vorticity-decomposition techniques is presented. A simplification of the TDM algorithm directly follows for the 3D vortex identification based on the *residual*-vorticity magnitude.

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