Topology of vortex creation in the wake of a circular cylinder

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The Karman vortex street behind a bluff body is an important situation where the flow is dominated by discrete vortices. Karman's analysis of the vortex street is also an early success of modeling discrete vortex structures with point vortices. In this paper we are concerned with the genesis of the vortex street, which, for a circular cylinder, takes place for a Reynolds number of about 50.

To describe the vortex street topologically, there are two obvious levels to consider: The instantaneous streamlines and the vorticity. Here we apply a full numerical simulation of the Navier-Stokes equation which we subsequently analyze with topological methods.

On the streamline level we find that for most of the Re range where the flow is periodic and two-dimensional, the streamline topology goes through a series of bifurcations during a period as shown in figure 1.

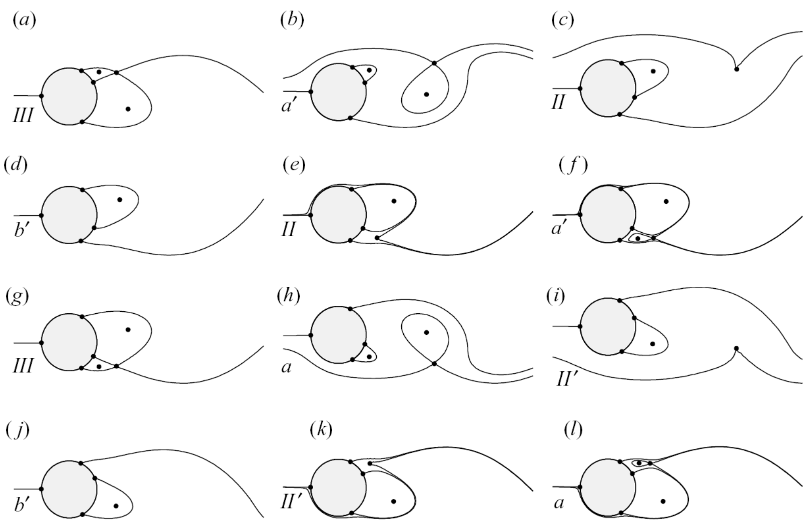

FIGURE 1: Topologies of the streamlines behind a circular cylinder for $Re = 100$ during one period. From [1].

It appears from figure 1 that on the streamline level, there are structures only very close to the cylinder. What may be identified as vortices created close to the cylinder surfaces are advected downstream and annihilated only after approximately one cylinder diameter.

Turning to the vorticity level, the picture is entirely different as shown in figure 2. Identifying the local minima and maxima of the vorticity as vortex centers, we see that the vortices

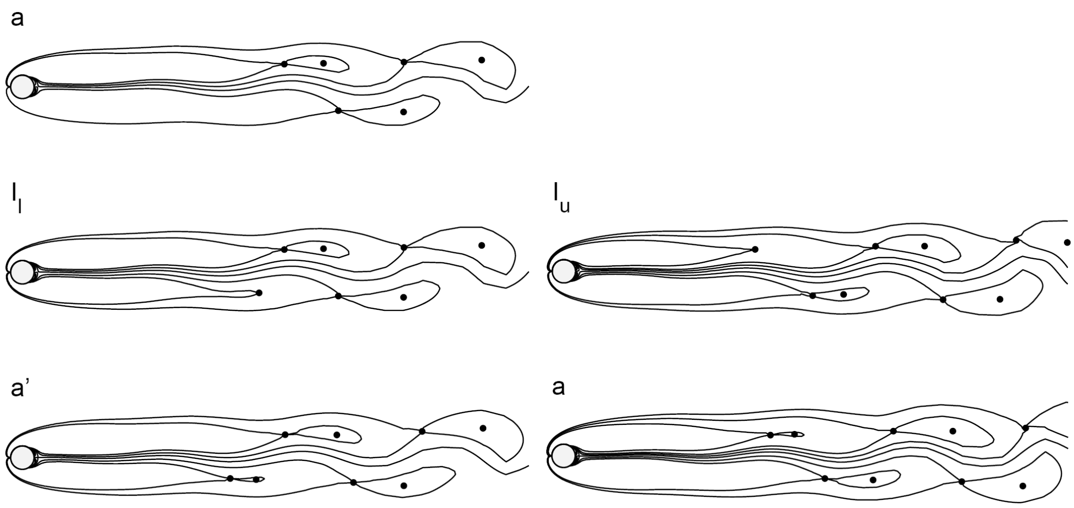

FIGURE 2: Iso-curves of vorticity during one period for $Re = 45.6$, right after the flow has turned periodic in the present simulations. From [2].

are created only quite far downstream. As the vortex closest to the cylinder is advected downstream, a new vortex are created (panels I_I and I_U.) After one period, all vortices are moved down one position. This picture persists for Re up to around 300. In the stationary phase, this means that there must be infinitely many vortices to all times, and, in particular, that these vortices are created exactly at the Re where the flow turns periodic in a Hopf bifurcation, stressing the truly global nature of the formation of the vortex street.

We propose that this topological view of the fields is fruitful for a systematic classification of more complex vortex streets.

References

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