Vortices for computing: the engines of turbulence simulation

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Abstract

Vortices have been described as the "sinews of turbulence" (Moffat *et al.*, 1994). They are also, increasingly, the computational engines driving numerical simulations of turbulence. In this talk I describe some recent advances in vortex-based numerical methods for simulating high Reynolds number turbulent flows. I focus on coherent vortex simulation (CVS), where nonlinear wavelet filtering is used to identify and track the few high energy multiscale vortices that dominate the flow dynamics. This filtering drastically reduces the computational complexity for high Reynolds number simulations. It also has the advantage of decomposing the flow into two physically important components: coherent vortices and background noise. In addition to its computational efficiency, this decomposition provides new insight into the structure and dynamics of high Reynolds number turbulence.

Vortex methods for the numerical simulation of fluid dynamics date back 80 years to Prager (1928) and Rosenhead (1931), but were not used commonly (or justified mathematically) until the work of Chorin (1973), Beale & Majda (1982) and others. Although they were originally developed for two-dimensional flows, a highly accurate three-dimensional vortex method for wake flows was introduced by Cottet & Poncet (2002). In vortex methods the circulation of the initial vorticity field is first discretized onto a set of N particles (or point vortices). The particles move by Lagrangian advection due to the velocity field of all other particles. Vorticity diffusion is modelled by redistributing circulation amongst nearby particles at the end of each time step. Although the particles are indeed the "engines" of the numerical simulation, they do not correspond precisely to the coherent vortices seen in direct numerical simulations and laboratory experiments. In other words, this method does not cut away the noisy "fat" from the coherent "sinews" of turbulence.

In order to reduce the computational complexity of numerical simulations of high Reynolds number turbulence Farge *et al.* (1999) proposed coherent vortex simulation (CVS) where the computational elements represent precisely the *coherent* vortices of the flow. To avoid the long-standing problem of how to define the coherent vortices in a turbulent flow we used the following computationally inspired ansatz: the coherent vortices are defined to be what remains once the (Gaussian) noise has been removed. Since noise is by definition incoherent, the remainder should correspond to what we intuitively think of as coherent vortices. These coherent vortices form the computational elements of the simulation: the wavelet modes are adapted at each time step to identify and track the coherent vortices. The effect of the noise is either modelled simply, or neglected entirely. Denoising is achieved by nonlinear wavelet filtering, since this procedure optimally removes additive Gaussian noise. Figure 1 shows an example of this approach applied to two-dimensional turbulence. It demonstrates that *all scales* of the coherent vortices are captured by only 0.3% of the total possible wavelet modes. The coherent vortices represented by the significant wavelet modes are both the "sinews" of the turbulence and the "engines" of the numerical simulation (since they are the computational elements, equivalent to the point vortices of vortex methods).

In addition to providing accurate and efficient numerical simulations, CVS also allows us to investigate the dynamics and structure of turbulence in new ways. For example, by extending CVS to the space–time domain Kevlahan *et al.* (2007) were able to estimate, for the first time, that the number of space–time modes of a two-dimensional turbulent flow scales like $Re^{0.9}$ (compared

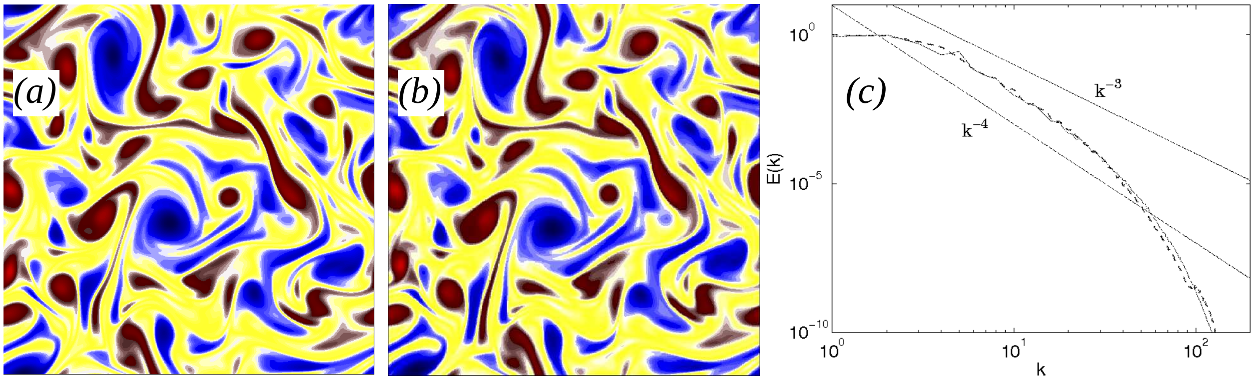

Figure 1: Vorticity field of two-dimensional turbulence at $Re = 40\,400$. (a) Computed from 263 169 Fourier modes using the pseudo-spectral method, (b) Computed using 7 895 coherent wavelet modes, (c) energy spectra: - - -, wavelet, — pseudo-spectral.

with the usual estimate of $Re^{1.5}$), while the number of spatial coherent modes scales like $Re^{0.7}$ (compared with the usual estimate of Re^1). These scaling exponents are a direct measure of the intermittency of the flow, and hence the space-fillingness of the coherent vortices. In my talk I will describe this work, and suggest how coherent vortex-based simulation techniques could lead to other new insights into physics of high Reynolds number turbulence.

In addition, I intend to present new results on how the spatial modes of three-dimensional turbulence scale with Reynolds number. The usual estimate is $Re^{9/4}$, however the high Reynolds number simulations of Yokokawa *et al.* (2002) suggest the actual exponent is significantly smaller. How much smaller it is will determine whether adaptive coherent vortex-based methods could, in principle, directly simulate the very high Reynolds number flows typical of engineering and geophysical applications.

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