Super-Rotation Flow in a Precessing Sphere

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The super-rotation here means that the majority of fluid inside a precessing sphere rotates around the precession axis with angular velocity larger than that of the precession rotation itself. This interesting phenomenon observed experimentally and numerically in our laboratory is explained to be driven by a cooperative interplay, in the boundary layer, between the Coriolis force, the pressure gradient and the spherical geometry.

Precessing Sphere We consider the motion of an incompressible viscous fluid confined in a precessing spherical cavity of which both the spin and precession angular velocity vectors are constant in magnitude and orthogonal to each other at all the time (figure 1). In the precession frame the fluid motion is described by the Navier-Stokes equation with the Coriolis term, the continuity equation and the non-slip boundary condition (given by the spin rotation on the spherical surface). Apart from the initial condition the flow dynamics in this system is characterized by only two parameters, namely, the Reynolds number $Re = a{\mathit{\Omega}_s}^2/\nu$ and the Poincaré number $\mathit{\Gamma} = \mathit{\Omega}_p/\mathit{\Omega}_s$. Here, a is the sphere radius, $\mathit{\Omega}_s$ is the spin angular velocity, $\mathit{\Omega}_p$ is the precession angular velocity, and ν is the kinematic viscosity of fluid. A variety of flow states, either steady, periodic or chaotic, are realized depending on these two control parameters. In this paper we focus our attention on the flow at large values of the Reynolds and Poincaré numbers, and present the numerical simulation results for $Re = 1000$ and $\mathit{\Gamma} = 10$.

Super-Rotation The details of the numerical method are omitted here, only saying that the boundary layer discussed below is numerically well resolved. The flow is described in the precession frame, in which neither the governing equations nor the boundary condition have explicit time dependence, implying that steady flows are possible relative to this frame. In figure 2 we show a schematic flow structure, suggested by steady flow realized for $Re = 1000$ and $\mathit{\Gamma} = 10$. The gray zone attached to the spherical surface is the boundary layer of thickness typically $0.05a$ (but the thickness is exaggerated in this figure). The rest, i.e. the white zone which occupies most volume of the spherical cavity, is the Taylor-Proudman region, where the flow field is almost uniform along the precession axis. The three closed lines indicate streamlines drawn by using the actual numerical simulation data. They rotate counter-clockwise around the precession axis, meaning that fluid particles swirl around this axis with angular velocity larger than that of the precession rotation. This is the super-rotation.

Eastward Drift The flow at the edge of the boundary layer drives the fluid motion in the Taylor-Proudman region. In figure 3 we show a typical fluid particle trajectory in the boundary layer by (a) the Mercator projection and (b) by a perspective view. Here, the solid lines represent the trajectory and the thin line in (a) is the mirror image of its southern ($z < 0$) part with respect to the equator. It is seen that the particle drifts slowly, $O(1/\mathit{\Gamma})$, toward East (increasing φ) while swirling clockwise (or in the same sense as the spin rotation). Observe that the northern part is slightly larger than the the mirror image of the southern part. This asymmetry with respect to the equator is responsible to the Eastward drift of fluid particles.

Boundary Layer Analysis Since the flow is driven by the spin rotation, it is nearly a solid-body rotation in the close vicinity of the spherical surface. The center of the swirling motion is located at the poles of the spin axis on the surface. As separated from the surface, the center at the positive pole shifts to the southwest, whereas the one at the negative pole to the northwest. As will be reported in the symposium, this shift is explained by a cooperative interplay of the Coriolis force, the pressure gradient, and the spherical geometry of the boundary. The Eastward drift followed by a spiral expansion (not shown here) of fluid particles in the boundary layer is represented in terms of the boundary layer solution for large Re and Γ.

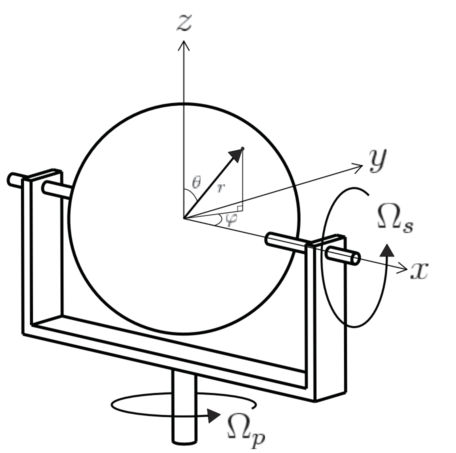

Figure 1: A precessing sphere with the Cartesian and spherical polar coordinates.

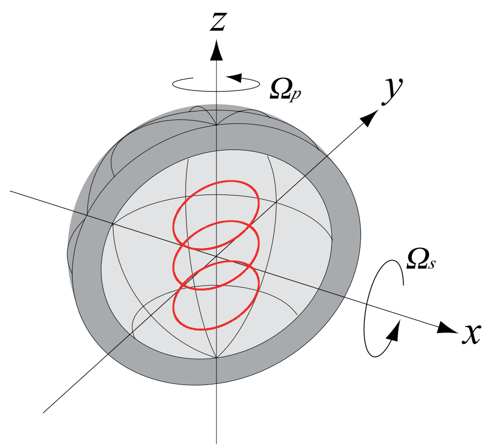

Figure 2: Boundary layer and Taylor-Proudman region.

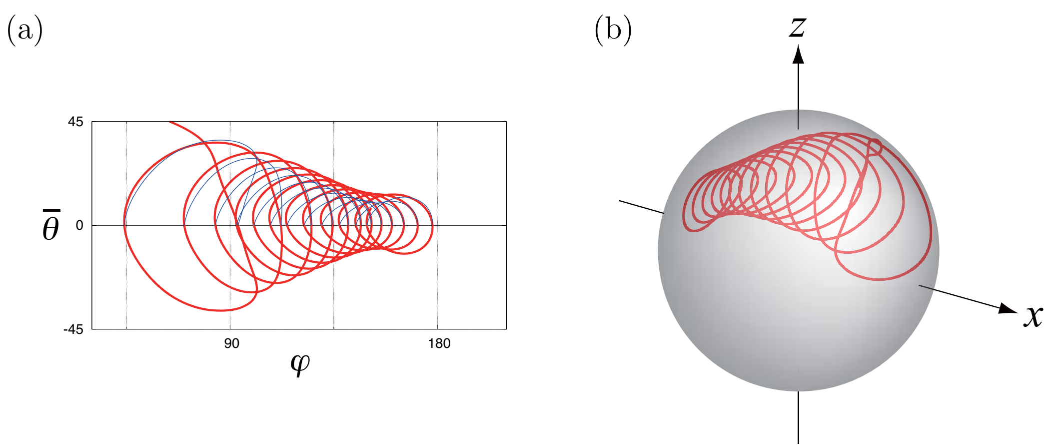

Figure 3: Particle trajectory. (a) Mercator projection. $\overline{\theta} \equiv \pi/2 - \theta$. (b) Perspective view. The particle moves from left to right in (a), whereas from right to left in (b).