Bifurcations in the wake of axisymmetric objects.

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I. NUMERICAL RESULTS

The flow of a viscous fluid past solid objects is characterised by successive bifurcations leading to various kinds of shedding modes. We use numerical simulation to study the case of fixed bodies with axial symmetry[1]. The case of a sphere is well documented[2]. A first bifurcation occurs for $Re \approx 210$ (where Re is a Reynolds number based on the body diameter and the incoming velocity), and gives rise to a steady shedding (SS) mode characterised by a reflexional symmetry (figure 1*a*) and a constant lift force. A second bifurcation takes place for $Re \approx 270$. The resulting mode (figure 1*b*) was called a reflexion symmetry preserving mode (RSP) because it is characterised by a reflexional symmetry and a lift force periodically oscillating about a non-zero mean value. A different picture is observed in the case of a flat disk. A first bifurcation is found for $Re \approx 116$, leading to a steady-state mode similar to that observed with a sphere. However the second bifurcation, which occurs for $Re \approx 121$, leads to a reflexion symmetry breaking mode (RSB, figure 1*c*). This mode mode is characterised by a lift force which periodically oscillates around a mean direction. A third bifurcation occurs for $Re \approx 139$, leading to a shedding mode which recovers the reflexional symmetry (figure 1*d*). This mode was called a standing wave (SW) mode by analogy with other applications.

II. NORMAL FORM THEORY

We introduce a theoretical model based on normal form theory to explain these findings. According to the central manifold theorem, the whole dynamics can be reduced to a system of coupled equations governing the amplitudes of the dominant modes of the problem. This system can be further reduced to its so-called *normal form* by taking into account the symmetries of the problem. Here, the dominant modes, revealed by linear stability analysis of the axisymmetric solution [3], are a steady-state mode (characterised by a complex amplitude a_0) and a Hopf mode (characterised by two complex amplitudes a_1, a_2). The symmetries of the problem correspond to the $O(2)$ symmetry

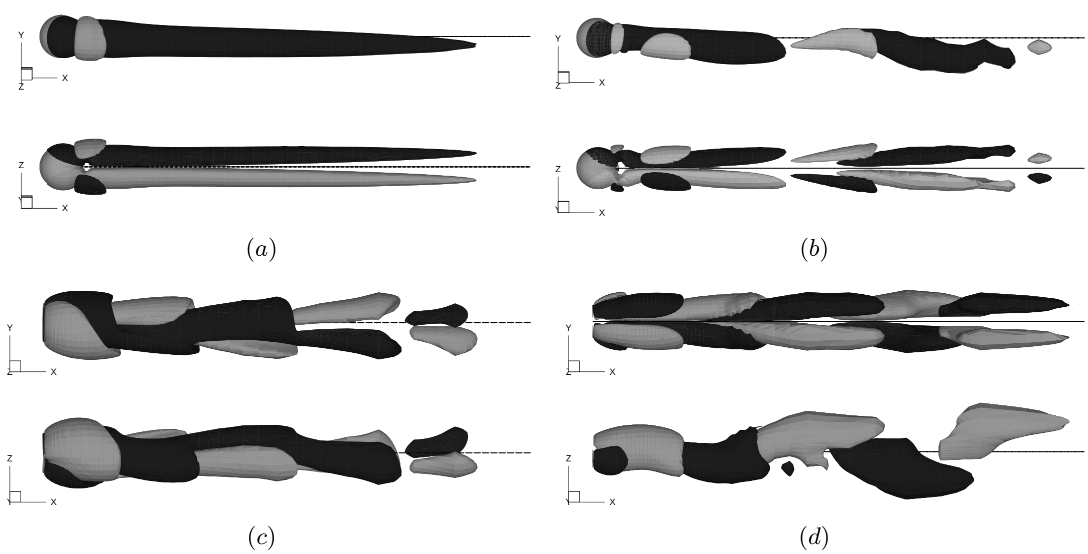

FIG. 1: Shedding modes in the wake of axisymmetric objects (axial vorticity contours) : (*a*) steady-state (SS) mode for a sphere with $Re = 250$, (*b*) reflexion symmetry preserving (RSP) mode for a sphere with $Re = 280$. (*c*) reflexion symmetry breaking (RSB) mode for a flat disk with $Re = 123$, (*d*) standing wave (SW) mode for a flat disk with $Re = 150$.

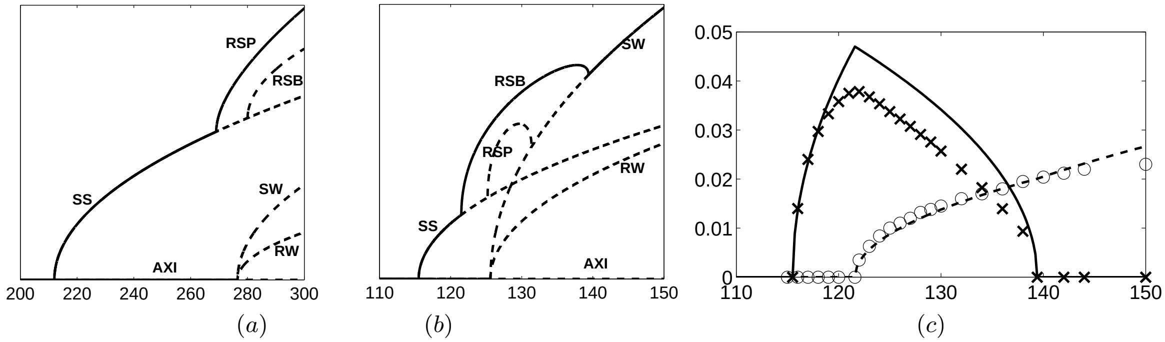

FIG. 2: Theoretical bifurcation diagrams for the sphere (a) and the flat disk (b). Solid (resp. dashed) lines denote stable (resp. unstable) branches. (c) : Lift coefficients of a flat disk *vs. Re*. Solid line : C_y (model); crosses : C_y (DNS); dashed line : $max(C_z)$ (model); Circles : $max(C_z)$ (DNS).

group (i.e. invariance by rotations and reflexions).

The *normal form* relevant to this situation is given in [4], and can be set into the following form:

$$\dot{a_0} = \lambda_s a_0 + l_0|a_0|^2 a_0 + l_1(|a_1|^2 + |a_2|^2)a_0 + il_2(|a_2|^2 - |a_1|^2)a_0 + l_3\bar{a_0}\bar{a_2}a_1, \tag{1a}$$

$$\dot{a_1} = (\lambda_h + i\omega_h)a_1 + \left(B|a_1|^2 + (A+B)|a_2|^2\right) a_1 + C|a_0|^2 a_1 + Da_0^2 a_2, \tag{1b}$$

$$\dot{a_2} = (\lambda_h + i\omega_h)a_2 + \left(B|a_2|^2 + (A+B)|a_1|^2\right) a_2 + C|a_0|^2 a_2 + D\bar{a_0}^2 a_1. \tag{1c}$$

This system was initially introduced to explain the various patterns observed in the Taylor-couette experiment which, under appropriate hypotheses, admits the same symmetry group. The simplest solutions of this system are a steady-state solution (SS), two "pure" Hopf modes including a standing wave (SW), and a rotating wave (RW, not observed here), as well as two "mixed modes" which can be identified with our RSP and RSB modes, respectively. We have selected values of the parameters entering the normal form which allow to obtain bifurcation diagrams qualitatively consistent with the numerical observations. Such diagrams are plotted in figure 2*a* for the sphere and 2*b* for the disk.

In the latter case, the model was also used to predict the lift forces exerted on the body. The corresponding lift coefficients are plotted as function of the Reynolds number and compared to the DNS results in figure 2*c*, showing excellent qualitative agreement. Note that a determination of the coefficients of the model has also been performed in [5] through a global stability analysis, and provide a further confirmation of the validity of the demarch.

III. DISCUSSION

The normal form introduced here has thus proved to be sucessful to explain the observed differences in the wakes of a fixed disk and a fixed sphere. This approach is also applicable to the wakes of axisymmetric bodies of any particular shape, and opens new ways to explore such problems. In particular, applying the present theory to moving bodies could be of great interest, the most challenging situation being that of freely-moving bodies whose motion is driven by buoyancy, such as rising bubbles. The path of such bodies is known to exhibit a variety of forms, including planar zigzags, helices, tumbling motions, etc. It is now recognized [6] that such path instabilities are directly linked to an instability of the recirculating region in the near-wake of the body. Extending the present approach to such problems in which the geometrical degrees of freedom of the body are intimately coupled to the wake dynamics is an extremely challenging question to which we will devote future efforts.

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