# Statistical Mechanics of Shear Layers

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This work addresses two issues:

1. It is known that statistical mechanics of point vortices describes surprisingly well the averaged velocity profiles of self-similar mixing layer. After development of statistical mechanics of vortex lines [1]- [5], where a vortex does not remain straight and is allowed to take quite wavy shapes in the course of motion, it appeared a concern that the above-mentioned feature of statistical mechanics of point vortices can be lost in three-dimensional theory. Indeed, consider a flow of ideal incompressible fluid between two parallel walls, $y$ is the coordinate normal to the walls, $-h \leq y \leq h$, $2h$ the distance between the walls. The flow is modeled by motion of a large number of vortices. The averaged velocity has the only non-zero component, $u$, that is parallel to the walls; $u = u(y)$.Assuming that vortices have the same intensity and the total discharge is zero, one obtains for the averaged stream function, $\psi = \psi(y)$ $(u \equiv d\psi/dy)$, the equation

$$\frac{d^2}{dy^2}\psi = -\sigma f(y), \qquad \left.\frac{d}{dy}\psi\right|_{y=\pm h} = \pm U, \tag{1}$$

where $f(y)$ is the probability to find a vortex at the point $y$, $\sigma$ the total vorticity.

In the case of point vortices,

$$f(y) = \frac{e^{-\beta\sigma\psi(y)}}{\int_{-h}^{h} e^{-\beta\sigma\psi(y')}dy'}, \tag{2}$$

and equations (1), (2) form a closed system of equations. This system can be solved analytically. Parameter $\beta$ may be viewed as determined by the initial energy of turbulent flow.

In the case of deforming vortex lines, $f(y)$ is expressed through the solutions of the eigenvalue problem,

$$\Delta\varphi - \beta\sigma\psi\varphi = -\lambda\varphi, \quad \frac{d\varphi}{dy} = 0 \text{ at } y = \pm h, \tag{3}$$

$\lambda$ being the minimum eigenvalue. Similarly to quantum mechanics, $f(y)$ is proportional to the squared solution of the eigenvalue problem:

$$f(y) = \frac{\varphi^2(y)}{\int_{-h}^{h} \varphi^2(y')\,dy'}. \tag{4}$$

There is no reason to expect that the velocity profiles found from the two quite different system of equations, (1)-(2) and (1),(3),(4), coincide. Nevertheless, this turns out to be the case: the velocity profiles are practically indistinguishable. More precisely: for each $\beta$ from

"3D problem" (1),(3),(4) there is $\beta$ from "2D problem" (1)-(2) for which the velocity profiles practically coincide. This might be an indication that for such flows three-dimensionality does not play an important role for the averaged velocity profiles.

2. For 2D jets, the statistical mechanics of point vortices does not describe the averaged velocity profiles correctly. There is a hope that the incorporation of three-dimensionality may decrease the discrepancies. This work is in progress now and is expected to be finished by the time of the conference.

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