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Title: Platonic and Archimedean solid based point vortex equilibria on the sphere

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Abstract: We consider the problem of finding relative equilibria for the N-vortex problem on a sphere. Using the five Platonic solids and thirteen Archimedean solids as basic building blocks, we find all vortex strengths associated with point vortices placed at the vertices for which the system of interacting particles is a relative equilibrium. The method is based on finding the fixed points of the nonlinear dynamical system governing the $N(N-1)/2$ equations for interparticle distances. We show how this leads to a basic question in linear algebra. Solve $A\vec{\Gamma} = 0$, where $\vec{\Gamma} \in R^N$ is the vector of vortex strengths, and $A \in R^{M \times N}$ is a rectangular, non-normal ($AA^T \neq A^T A$) 'configuration' matrix determined by the particle positions. Configuration matrices with non-trivial nullspaces ($\det(A^T A) = 0$) yield equilibria for particle strength vectors $\vec{\Gamma} \in Nullspace(A)$. Our main tool for finding and classifying equilibria is the singular value decomposition of A. The eigenvectors of the covariance matrix $A^T A$ corresponding to the zero singular values form a basis for the nullspace of A and thus are used to construct $\vec{\Gamma}$. The (normalized) distribution of all the non-zero singular values yields the Shannon entropy of the configuration, which we interpret as the level of 'disorder' of the system. First, we obtain the singular value decomposition of the five Platonic solids and use it to obtain a basis set for the nullspace of A. Then we show that only two of the thirteen Archimedean solids form equilibria for any vortex strengths. By taking superpositions of the Platonic and Archimedean solids, we construct families of new (symmetric) equilibria. We also show how asymmetric equilibria, and symmetric configurations with defects can be generated by allowing a subset of the points in the symmetric lattice to execute a random walk on the sphere, while the other points are held fixed. In this way, we characterize lattices with defects via matrix perturbation theory. Joint work with P.K. Newton.

References:

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