Statistical theory applied to a vortex street generated from meander of a jet

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Key words: vortex street, Kármán vortex, meander of jet, potential vorticity, barotropic instability, statistic theory, mixing entropy

In winter in East Asia Region, cold airmass often outbreaks from the Continent. Under such weather conditions, in the lee side of an island with a high mountain, a weak wind region is made and a vortex street called Kármán vortex is often generated and its flow pattern is visualized by clouds. Similar vortices are also found in the oceans: The Kuroshio current after leaving the coast of Japan, which is called Kuroshio Extension, usually meanders strongly and generates eddies on its both sides. In this way, jet or weak flow region often meanders because of its instability and generates a vortex street, in the atmosphere and oceans.

In order to investigate the general features of the formation process and statistics of vortex streets generated from the meander of jets, a simple model described by barotropic quasi-geostrophic equation (aka CHM equation) is numerically time-integrated, and the vorticity distribution in the mature steady state is investigated.

To describe this steady state, the statistic theory for vorticity, which was proposed by Robert and Sommeria (1991), is a very powerful tool. According to their theory, the final steady state is determined by a simple principle familiar in statistical physics: in the final equilibrium state, the 'mixing entropy' must be maximum under the conservation restriction for several quantities such as energy, momentum and so on. They considered situations where the initial state consists of uniform vorticity 'patches' of finite kinds of vorticity values, and derived the relation which the vorticity and the stream-function must satisfy. As an example of the application of this theory, Sommeria et al. (1991) calculated the simplest situation consisting of two kinds of uniform vorticity patches, which describes a shear flow, and showed the validity of their theory.

A jet-shaped current situation demands at least three kinds of uniform vorticity patches. This situation was investigated by Thess and Sommeria (1994) and they showed that this 'maximum mixing entropy principle' is also applicable for a trapezoidal jet situation. If the initial paralell flow field oriented to x-direction is divided into zonal regions with three kinds of potential vorticity values $-Q_0$, Q_0 and 0, their statistic theory predicts that the stream function ψ and the potential vorticity q of the steady equilibrium state, must have relation

$$q = \frac{2Q_0 \exp\gamma \sinh[Q_0(\alpha y - \beta\psi)]}{2\exp\gamma \cosh[Q_0(\alpha y - \beta\psi)] + 1},$$

where, α, β, γ are constants. The vortex pair must be separated to the borders of the region, and it was confirmed by the numerical calculation.

The numerical calculation in the study here, the initial current is a quasi-geostrophic triangular jet with potential vorticity distribution $-Q_0$ for $(-L < y < 0)$, Q_0 for $(0 < y < L)$ and 0 for $(|y| < L)$. However, the vortex pair formed by its instability remains near the original jet current with a moderate distance, not separated to the borders of the region. The scatter plotting of stream-function ψ vs potential vorticity q in this calculation shows that the relation between these two quantities still obey the relation derived by Thess and Sommeria (1994). Nevertheless, the relation is not described by only one set of parameters, but it separates to two distinct 'branches'. One of these two branches corresponds to the region near the initial jet where the mixing fully occurs, and the other to the both sides of the jet which the mixing does not reach, and the potential vorticity q is almost 0 on this branch.

Since two branches cross at the origin of the ψ-q plane, the two branches are not clearly separated. In order to distinguish these two branches, we consider a distribution of passive scalar c which is distributed as C_0 for $(y < |L|)$ and 0 for $(y > |L|)$. In the equilibrium state, stream function ψ and scalar c must have relation

$$c = \frac{2C_0 \exp\gamma \sinh[C_0(\alpha y - \beta\psi)]}{2\exp\gamma \cosh[C_0(\alpha y - \beta\psi)] + 1}.$$

The scatter plotting of ψ vs c of the numerical calculation shows separation between these two branches much more distinctly, and we can identify how far extends the region where the mixing occurred.