On the instability of elliptic hetons

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Classic solution by Kirchhoff [5] of the problem of elliptic vortex patch (having the semi-axes a and b) with a constant vorticity ω rotating as a solid body with constant angular velocity $\Omega = \omega ab/(a^2 + b^2)$ has been known since 1876. Six years later, Love [10] showed that this stationary solution would become unstable at $\chi = a/b > 3$. More later Kirchhoff solution underwent numerous generalizations, in particular:

- Chaplygin [1] (see also [11]), and later Kida [4], Dhanak & Marshall [2], Legras & Dritschel [9] and others showed that taking into account the external velocity field, depending linearly on coordinates (what is analogous to an affine transformation of coordinates converting ellipses into ellipses), allows the construction of a solution which describes the behaviour of a pulsing vortex, revolving with variable velocity and changing with time.

- Polvani & Flierl [12] introduced the notion of a "generalized Kirchhoff vortex" for a system of N embedded elliptic patches. They investigated stability of such solution and constructed diagrams of stable and unstable states in the space of external geometric parameters.

- Kozlov [6, 7] generalized the problem of elliptic vortex in taking into account the effect of "entrainment" due to introduction of "effective" bottom friction. This mechanism initiates cyclonic-anticyclonic asymmetry observed in ocean and atmosphere conditions. Conditions under which the system provides the Kirchhoff, Chaplygin and Kida solutions are given in [7].

In the present work we examine the problem of a two-layer elliptic vortex evolution in a stratified rotating fluid having two non-mixing layers. It is supposed, that at the initial moment, the shape of elliptic vortices of the upper and bottom layers is the same, and they are located strictly one under the other, but they have opposite signs of ω. So, a two-layer vortex represents a heton [3]. For the case of $\chi = 1$, a stationary solution in the form of an axisymmetric two-layer vortex with a vertical axis has been studied in [8, 13] for stability with respect to small harmonic disturbances of their contours. It is shown, that when $\gamma = R/R_d > 1.7$ (R is a characteristic radius of the vortex patch, and R_d is the internal Rossby radius of deformation), there is a possible appearance of unstable modes.

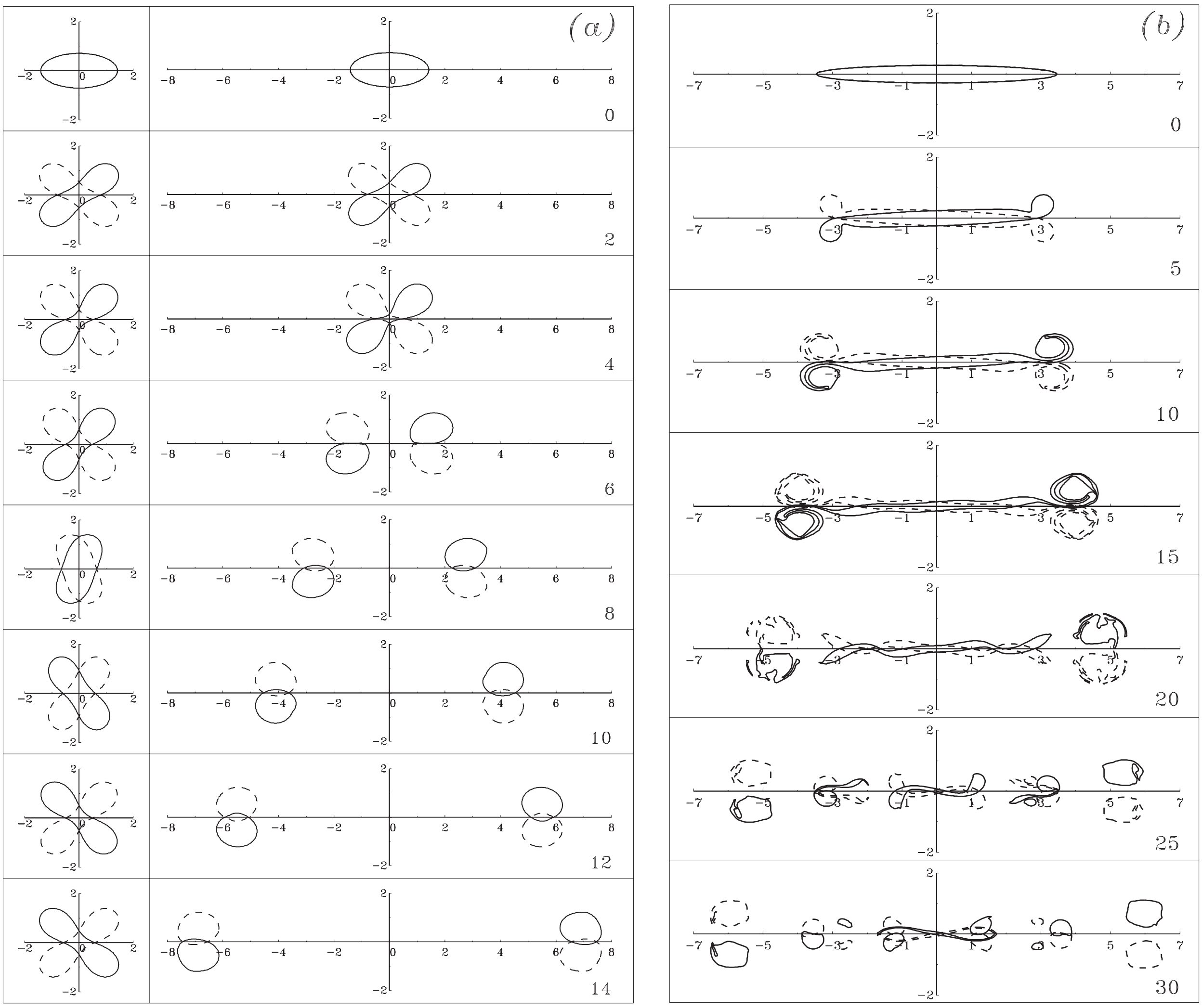

Figure 1: Evolving contours of vortex patches of the upper (solid line) and lower (dashed line) layers in the indicated moments of non-dimensional time. *(a)* Synchronous configurations of the stable (left, at $\gamma = 1.56$) and unstable (right, at $\gamma = 1.57$) elliptical hetons at $\chi = 2$; *(b)* Example of the cascade instability of elliptical hetons at $\gamma = 2.5$ and $\chi = 12$.

At $\chi > 1$, investigation for stability is possible only by numerical methods. In this work, basing on the method of contour surgery, we have constructed diagrams of all possible states of initial elliptic two-layer heton in the space of variables (γ, χ). Domains of existence were obtained both for neutrally stable solutions, and for solutions with different types of instability. Some examples are given in Figure 1.

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