LOCALIZED SHALLOW-WATER DIPOLES

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Solutions for steadily translating localized dipolar vortical structures, or modons, are sought in the 1½-layer rotating shallow water (RSW) model. The focus is on the smooth intense modons with circular separatrices, in which the potential vorticity (PV) field is continuous, and the pressure (hence, the active-layer thickness) and velocity are smooth, while the Rossby number and relative deviations of the active-layer thickness from the static level are of the order 1. Fundamental modon invariants are derived, and the boundary conditions at the separatrix, the streamline demarcating the trapped-fluid region and the flow outside it, are established. The problem is solved numerically using a Newton–Kantorovich procedure, Fourier–Chebyshev spectral expansion and collocations. We show (analytically) that the translation speed of any RSW modon is necessarily smaller than that of inertia-gravity waves, and (numerically) that, in smooth modons, an even stronger restriction is imposed by the requirement that the active-layer thickness should be positive. The maximal translation speed as a function of the modon size decreases monotonically and, for reasonable sizes, is appreciably smaller than the gravity-wave limit. As distinct from the circular quasi-geostrophic (QG) modons, the RSW modons with circular separatrices display nonlinearity of the potential vorticity (PV) vs. streamfunction relation, and the cyclone–anticyclone asymmetry: the cyclone is more intense and compact than the anticyclone.

Background

Anti-symmetric dipolar two-dimensional flows represent a classical example of a modon. The first solutions that can be categorized as modons were suggested by Lamb (1895), Chaplyging (1903) (see Meleshko & van Heijst, 1994, where this solution is reproduced and discussed in detail), and by Stern (1975) and Larichev & Reznik (1976). The interest in modon solutions was stimulated by the extensive research of mesoscale oceanic and atmospheric eddies in the 70s and 80s. The importance of modons was evidenced, for example, by the discovery of the so-called mushroom structures, quite durable modon-like currents common in a wide range of space scales of ocean currents (Fedorov & Ginzburg 1984); vortical structures in magnetized plasma represent another field of application of modons.

During the past three decades, a considerable progress has been made in developing the modon theory in application to rotating and stratified fluids within the QG approximation (see, e.g., Kizner, 1997; Kizner et al. 2003 and references therein). Quasi-geostrophicity means that the Rossby number (that, in modons, can be defined as U/Lf, where U and L are the modon's translation speed and size, respectively, and f is the Coriolis parameter) is small, and the relative deviations of isopycnal surfaces (or of the layer interfaces in a layered model) from their static levels, are also small. In intense vortices, both conditions are normally violated. Moreover, the observed mushroom structures are marked by asymmetry. These facts motivate us to look for modons in a 1½-layer RSW model, which is free of the above restrictions. In this model, two fluid layers of different densities are considered, one of them being infinitely deep and, therefore, passive. The variation in the thickness of the other, active layer is allowed to be comparable to the layer thickness itself. A barotropic interpretation of this model is a single free-surface fluid layer. So far there were only a few indirect numerical and experimental indications of the possibility of the existence of RSW modons. Our research is devoted to the construction of the long awaited explicit RSW modon solutions on the f-plane (where f = const) and to the clarification of the conditions allowing their existence.

Results

When a traveling modon is considered in a co-moving frame of reference, two areas can naturally be distinguished, the interior (or trapped-fluid) region where the streamlines are closed, and the exterior region where they are open. We assume that the closed streamline demarcating the two regions, the separatrix, is a circle. First the exterior flow is found, and the boundary conditions for the interior problem are established. The pressure and velocity are required to be continuous, whereas PV can, generally, be discontinuous at the separatrix. Smooth solutions marked by the PV continuity at their separatrices are of the utmost interest due to their potential stability (cf. Kizner et al. 2003).

We provide analytical arguments in favor of the existence of localized steadily translating vortical structures and, using a specially designed numerical algorithm, produce a broad spectrum of RSW modons. Smooth modons constitute a two-parameter family, the parameters being L (the separatrix radius) and U. Like QG modons, the simplest RSW modon is a pair cyclone–anticyclone. The ageostrophicity is apparent primarily in the cyclone–anticyclone asymmetry, the cyclone being more compact and having a stronger peak PV than the anticyclone (Fig. 1 a, b). Also, as distinct from the circular QG modons, the dependence between PV and the co-moving streamfunction Ψ is nonlinear. With increasing U and L the modon asymmetry enhances, the minimal thickness of the active layer (in the cyclone center – see Fig. 1 c) decreases, and the nonlinearity of the Bernoulli function and PV vs. Ψ relations in the interior region becomes better pronounced (Fig. 1 d). The requirement that the thickness of the active layer in the cyclone must be positive imposes a constraint on parameters U and L.

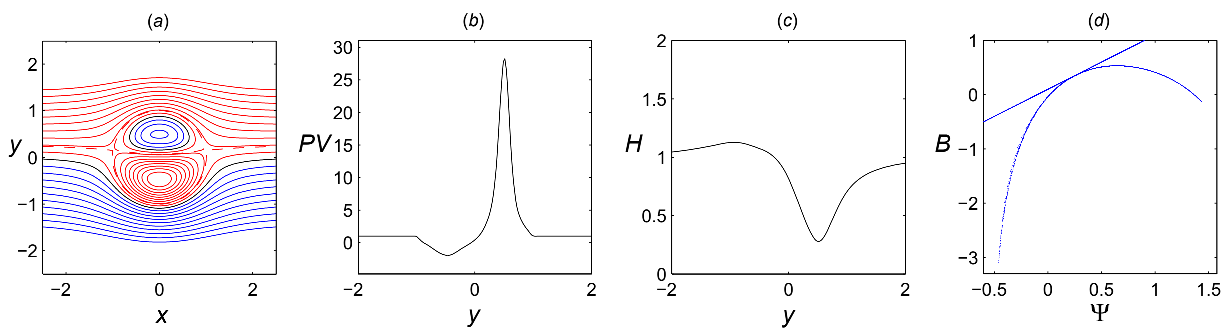

Fig. 1. An RSW modon of radius 1 traveling along the x-axis (non-dimensional variables). (a), co-moving streamfunction Ψ; red, positive contours; blue, negative contours; dashed, separatrix streamline; smaller vortex, cyclone; bigger vortex, anticyclone. (b), PV cross-section at $x = 0$. (c), active-layer thickness H at $x = 0$. (d), Bernoulli function B vs. Ψ relation; straight line, exterior region; curved line, interior region.

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