Falling, flying, swimming, flapping, ...: understanding fluid-solid interactions using a vortex shedding model

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ABSTRACT

We investigate the fluid-solid interaction problem of the motion of thin solid bodies in high-*Re* non-turbulent flows. A reduced-order two-dimensional model is proposed for the fluid motion using potential flow theory. Point vortices with monotonically increasing intensity are shed from the sharp edges of the solid to enforce the regularity condition on the flow. Several applications are presented from the analysis of biological locomotion in fluids to the study of falling cards or flapping flags.

The interaction between the motion of a solid body and the surrounding fluid is an essential element for non-terrestrial animal locomotion: insects or fishes flap their wings or fins to create around them a highly unsteady flow able to generate the lift and thrust forces necessary to move or hover in the air or water. This interaction is also responsible for the chaotic-looking motion of a falling paper card in the air or the flapping of flags in strong winds. The full numerical solution of the coupled solid-body motion is computationally expensive as it requires solving a system of partial differential equations (the Navier–Stokes equation) coupled on the moving boundaries to a system of ordinary or partial differential equations (Newton's law or internal solid dynamics). Proposing simplified numerical and physical models for this complex problem is a challenging and ongoing research topic.

The applications mentioned above are characterized by slender solid profiles and high Reynolds number *Re*. The effect of viscosity is concentrated in thin boundary layers that separate in free shear layers due to the unsteady motion of the trailing edge relative to the fluid, and roll-up into strong vortices. We focus here on two-dimensional problems without separation and choose to represent the fluid motion using potential flow theory. The vortex formation is represented by the shedding of point vortices with monotonically increasing intensity. The intensity of these vortices is adjusted at all time to satisfy the regularity of the flow at the solid body edges, and their velocity is given by the Brown–Michael equation that conserves the fluid momentum in an integral sense around the vortex and the branch cut linking it to the generating corner. The flow is entirely determined by the vortex properties (intensity and position) and the solid velocity, and the pressure at the surface of the body can be computed using complex analysis. We focus here on thin solid bodies, using conformal mapping or a bound vortex sheet representation for respectively rigid and flexible bodies.

In a first step, the kinematics of the solid is prescribed and the resulting pressure forces are computed. Qualitative and quantitative physical insights are obtained on biological locomotion in fluids using simple rigid or flexible flapping schemes: structure of the wake and drag/thrust transition, efficiency of the flapping,... (see Figure 1). The low computational cost of this method and its ability to capture the main physical characteristics of the problem makes it particularly well-suited for optimization problems (of the solid kinematics for example) for which the cost of full numerical simulation is prohibitive.

In a second step, introducing the solid dynamics (as well as its internal response for flexible objects), the fluid and solid problems are coupled, and the effect of an outside forcing on the coupled system is studied: fall of a rigid plate under the influence of gravity (see Figure 2), flapping of a flexible flag under the influence of the wind.



Figure 1: Drag/Thrust and propulsion efficiency of a wavelike flexible flapping scheme. v_{ϕ} is the wave-speed. (a) The flow at infinity U_{∞} and the flapping motion are prescribed, unsteady point vortices are shed from the trailing edge. (b) Influence of the flapping amplitude and frequency on the horizontal force. Red zones correspond to drag and blue zones to thrust. (c) Locomotion efficiency (ratio of the available thrust to the energy input \mathcal{P}).



Figure 2: Trajectory of a falling rigid plate with inertia ratio $Fr = M/2\rho l^2 = 0.7$ released with an angle θ_0 with the horizontal, obtained using the unsteady point vortex method. The vortex shedding destabilizes the position $\theta = 0$ and a transition to large scale fluttering or tumbling regimes is observed.