Motion of a ring structure of coherent vortices on a sphere with pole vortices

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An important aspect to be considered in the study of planetary atmospheric flows is that the fluid is confined on the surface of rotating sphere. As a simple mathematical model, we consider incompressible and inviscid flows on the sphere with two point vortices that are fixed at the north and the south poles of the sphere. In this model, interaction between coherent vortex structures plays an key role since vorticity is invariant along the path of a fluid particle. As a matter of fact, the pole vortices cannot represent the effect of the rotation rigorously since vorticity is no longer a conserved quantity when the rotation is taken fully into consideration. Nevertheless, the pole vortices could generate an outer flow that taken into account the local contribution of rotation of the sphere locally, and this pole-vortices model provides us with a simple Hamiltonian framework to treat the problem by analytic and numerical means. In the present talk, I would like to give a brief survey of recent research on the motion of coherent vortex structures such as vortex sheets and point vortices on the sphere with pole vortices.

First, we consider the evolution of a vortex sheet, which is a discontinuous surface of the velocity field in incompressible and inviscid flow. The vortex sheet is a mathematical idealization of shear flows that are often observed in atmospheric flows. Analytic and numerical research on the vortex sheet on the sphere have revealed that (1) the vortex sheet becomes linearly unstable due to Kelvin-Helmholtz instability, (2) its curvature diverges in finite time and (3) it rolls up into a doubly branched spiral, when the equation of motion of the vortex sheet is regularized by the vortex method [1]. Furthermore, after a long-time evolution, the regularized vortex sheet forms a structure consisting of tightly winding vortex spirals that are aligned along a line of latitude. Thus the Kelvin-Helmholtz instability gives rise to the ring structure of coherent vortex-spirals.

Next questions are whether the latitudinal ring configuration of the vortex spirals is stable or not, and how it evolves when it becomes unstable. In order to deal with the problem mathematically, we further approximate the tightly winding spirals with identical point vortices. The configuration of point vortices that are equally spaced along the line of latitude is known as the N-ring. With the linear stability analysis of the N-ring [2] and the systematic reduction method of low-dimensional systems in [4], we investigate the evolution of the unstable N-ring and show that it exhibits a chaotic behavior [3, 6, 7]. Moreover, we consider the background vorticity field corresponding to the solid-body rotation of the sphere [5] as another model for the effect of rotation. Then we numerically show how the N-ring interacts with the background vorticity field, and compare the numerical results with those for the pole-vortices model.

References

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