Asymmetric vortex merger: mechanism and criterion

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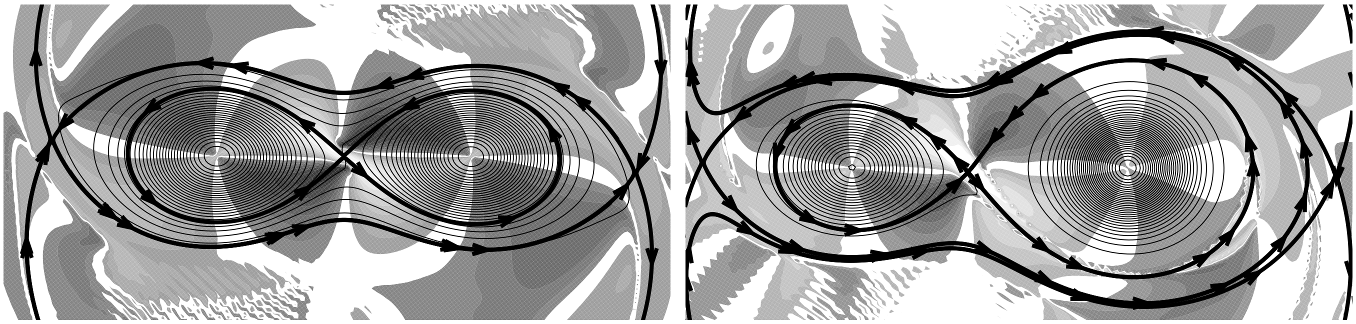
The merging of two co-rotating vortices is an elementary vortex interaction of both fundamental and practical significance. It plays an important role in the transfer of energy and enstrophy across scales in transitional and turbulent flows. It may also occur in the nearfield wake of an aircraft. Yet despite its apparent simplicity and much research, the physical mechanisms of vortex merger have not been fully understood [1, 2]. In the ideal case of *symmetric* vortex pairs, it is known that if the aspect ratio (core size/separation) a/b, exceeds a critical value, the vortices move towards each other and eventually form a single vortex. What is more commonly observed are *asymmetric* (unequal size and strength) vortex pairs. In this case, there is a greater range of flow behavior and the interaction of the vortices may result in the destruction of the smaller/weaker vortex. Previous studies of inviscid flow have identified a number of flow regimes [3, 4]. However, the associated physics are unclear and a general merging criterion has not been determined.

In this work, we investigate the merging of two co-rotating vortices in a viscous fluid. Asymmetric (equal size, unequal strength) vortex pairs are considered. The variations exhibited in these flows provide further insight into the interaction of vortices and enable the development of a more generalized description and criterion for merger.

Two-dimensional numerical simulations are performed for the study. The initial flow consists of two Lamb-Oseen co-rotating vortices of equal size and unequal strength (circulation, Γ). The initial aspect ratio is $a_o/b_o = 0.157$, where a_o is defined based on the second moment of vorticity. The Reynolds number of the stronger vortex is $Re_{\Gamma,1} = \Gamma_{o,1}/\nu = 5000$ and that of the weaker vortex varies in the range $0.4 \leq Re_{\Gamma,2}/Re_{\Gamma,1} \leq 1.0$. A convective time scale is the approximate rotational period of the system, $T = 2\pi^2 {b_o}^2/\overline{\Gamma}_o$, where $\overline{\Gamma}_o = 0.5(\Gamma_{o,1} + \Gamma_{o,2})$. Nondimensional time is ${t_c}^* = t/T$.

Our analysis of the symmetric vortex pair elucidates the key physical mechanisms of convective merger in terms of four phases of development [2]. We consider the flow in the co-rotating frame and describe the deformation of the vortices in terms of the interaction of vorticity gradient, $\nabla\omega$, and rate of strain, S (figure 1). In the diffusive/deformation phase, diffusion of the vortices establishes the interaction of $\nabla\omega$ and mutually induced S. As indicated in figure 1a, each vortex exhibits a quadrapole structure of $P_s = -(\nabla\omega^T \mathsf{S} \nabla\omega)/|\nabla\omega|^2$, corresponding to alternate regions of gradient amplification/attenuation by compressive/extensional straining, as it elliptically deforms. During this time, a distinct functional relation between ω and streamfunction exists suggesting quasi-equilibrium conditions. However, in the vicinity of the hyperbolic points, and in particular the central hyperbolic point (CH) where mutual interaction strengthens $\nabla\omega$ amplification (figure 1 - light shading near center is $P_s > 0$), the interaction of $\nabla\omega$ and S eventually produces a tilt in ω contours [1]. At the outer hyperbolic points, this initiates filamentation. During the convective/deformation phase, the induced flow by the filaments acts to advect the vortices towards each other and enhances the mutually induced S but does not drive the merger to completion. The enhanced tilting and diffusion of ω near the CH causes ω from the core region to enter the exchange band where it is advected away. This leads to the departure from quasi-equilibrium conditions. In the convective/entrainment phase, the vortex cores are thereby eroded and rapidly move towards each other as they are *jointly* entrained into the exchange band, whose induced flow becomes dominant and transforms the flow into a single vortex. A critical aspect ratio, associated with the start of the convective/entrainment phase, is determined for a range of flow conditions [2]: $(a/b)_{cr} = 0.235 \pm 0.006$. In the final diffusive/axisymmetrization phase, the flow



(a) $Re_{\Gamma,2}/Re_{\Gamma,1} = 1.0$ (b) $Re_{\Gamma,2}/Re_{\Gamma,1} = 0.6$

Figure 1: Streamlines in co-rotating frame (thick lines) and vorticity contours (thin lines) with gray shading corresponding to $\nabla\omega$ production term, $P_s = -(\nabla\omega^T \mathsf{S} \nabla\omega)/|\nabla\omega|^2$ (light gray scale: $P_s > 0$, dark gray scale: $P_s < 0$) for (a) $Re_{\Gamma,2}/Re_{\Gamma,1} = 1.0$, $t_c^* = 1.54$, (b) $Re_{\Gamma,2}/Re_{\Gamma,1} = 0.6$, $t_c^* = 0.34$.

evolves towards axisymmetry by diffusion.

In the case of the asymmetric vortex pair, the disparity between the vortices alters the flow structure and development (figure 1b). P_s is stronger at the weaker vortex due to the difference in induced S, and the tilt of ω contours and subsequent core detrainment occurs earlier than the stronger vortex, i.e., the onset of the convective/entrainment phase differs for each vortex. However, the dominant attracting motion occurs only when, *and if*, the stronger vortex reaches this phase since the induced flow by the weaker vortex is insufficient to advect the stronger vortex. We consider the onset of the core erosion process to be associated with the flow achieving a sufficiently high strain rate, with respect to some characteristic ω, for the process to proceed. Through scaling analysis, the ratio of the strain rate at the CH, S_{CH}, to the vorticity at the vortex center, ω, may be related to a and $d_{|CH-V_i|}$, which is the distance from the center of vortex i to the CH. For example, for vortex 2:

$$\gamma_2(t) \equiv \left(\left[\frac{S_{CH}}{4\,\omega_{v_2}} \right] \left[\frac{1}{\frac{1}{2}(1 + \frac{Re_{\Gamma,2}}{Re_{\Gamma,1}})} \right] \right)^{1/2} \propto \left[\frac{a_2}{2\,d_{|CH-V_2|}} \right] \left[1 + \frac{Re_{\Gamma,1}}{Re_{\Gamma,2}} \frac{d^2_{|CH-V_2|}}{d^2_{|CH-V_1|}} \right]^{1/2} \left[\frac{1}{1 + \frac{Re_{\Gamma,2}}{Re_{\Gamma,1}}} \right]^{1/2}. \quad (1)$$

Note that the strain parameter, γ_i, for symmetric vortices reduces to $\gamma_1 = \gamma_2 = (S_{CH}/4\,\omega_v)^{1/2} \propto a/b$. The critical condition corresponding to the start of the convective/entrainment phase for vortex i is $\gamma_i(t) = \gamma_{cr,i}$. For all our simulations, we find a single critical value for both vortices, i.e., $\gamma_{cr,1} = \gamma_{cr,2} = \gamma_{cr} \approx 0.246 \pm 0.005$, which is close to the values of $(a/b)_{cr}$ for symmetric pairs.

Based on the timing of the flow processes in our viscous flows, we characterize three distinct flow regimes that are comparable to those identified in the previous studies [3, 4]. Complete merger occurs if both vortices reach γ_{cr} and are entrained into the exchange band and jointly transform the flow into a single vortex. Partial merger occurs if both vortices reach γ_{cr} and are entrained into the exchange band; however, the weaker vortex core is destroyed during the process. The weaker vortex is strained-out when only the weaker vortex core is eroded and entrained into the exchange band, i.e., the stronger vortex does not reach γ_{cr}. Details will be presented in the full paper.

References

[1] Brandt, L. K. and Nomura, K. K. (2006). The physics of vortex merger: further insight. *Phys. Fluids*, 18:1-4.

[2] Brandt, L. K. and Nomura, K. K. (2007). The physics of vortex merger and the effects of ambient stable stratification. *J. Fluid Mech.*, 592:413-446.

[3] Dritschel, D. G. and Waugh, D. W. (1992). Quantification of the inelastic interaction of unequal vortices in two-dimensional vortex dynamics. *Phys. Fluids*, 4:1737-1744.

[4] Trieling, R. R., Velasco Fuentes, O. U. and van Heijst, G. J. F. (2005). Interaction of two unequal corotating vortices. *Phys. Fluids*, 17:1-17.