Visualizing Mixing in Geophysical Vortices

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Abstract

The structure of transport and mixing in vortical flows cannot be readily discerned by considering velocity or vorticity alone. However, the underlying Lagrangian coherent structures that govern transport in the flow can be extracted via computation of finite time Liapunov exponents. Visualizations of the Lagrangian coherent structures extracted in this way are provided for three geophysical vortices. We observe that for the apparently disparate cases of hurricanes and ocean eddies, the underlying transport mechanism in each case is identical and occurs via lobe dynamics.

Introduction. Over the last several years, various methods have been developed to study coherent structures in time-dependent flows. The primary goal of these methods is to identify the key structures and manifolds within the flow that govern transport and mixing [9]. Surprisingly, even ostensibly turbulent flows may exhibit simple structural skeletons that dictate how mixing occurs [6].

In this study, we apply a finite time Lyapunov exponent (FTLE) technique to extract Lagrangian coherent structures (LCS) in vortical geophysical flows. In particular, we study hurricanes in atmospheric flows and mesoscale eddies in oceanic flows. The analysis demonstrates that transport surrounding these geophysical vortices is largely characterized by lobe dynamics, a well-understood mechanism of geometric mechanics.

The method we apply was first proposed by Haller [2] and later developed by others including Lekien, Couillette, Shadden, Marsden, and Dabiri [10; 5]. In order to extract the LCS, we first compute the flow map by direct integration of the flow, and then compute the FTLE, a time-dependent scalar field represented by the symbol σ. If ϕ_t^{t+T} is the flow map from time t to time $t+T$, then the FTLE is given by

$$\sigma(x,t) := \frac{1}{|T|} \ln \left\| \frac{d\phi_t^{t+T}(x)}{dx} \right\|_2 . \qquad (1)$$

The LCS are defined to be ridges in the FTLE field and represent surfaces of locally maximal separation. These separatrices act as barriers to transport and delineate regions with different dynamical behavior. In practice, we compute both the forward-time and the backward-time flow maps in order to uncover repelling and attracting surfaces respectively. Plotting both the repelling and attracting structures allows us to see the direct relation to lobe dynamics theory for periodically-perturbed flows.

Atmospheric Vortices. A key requirement for the improvement of hurricane forecasting is enhanced understanding of the transport processes both into and out of the hurricane. For instance, Houze et al have noted that the entrainment of dry air tends to lead to destabilization of the hurricane eyewall and a decrease of hurricane intensity [3]. Although vorticity, humidity, and temperature fronts are correlated with LCS, they are not sufficient to capture sharp boundaries and the details of transport. In Figure 1, the LCS computed using NCEP/NCAR Reanalysis Data for the wind field in the Western Pacific at the 850 mb pressure level for the time period of Typhoon Nabi (September 2005) is shown. Despite the complex flow surrounding the typhoon, we readily observe that LCS accurately captures the boundary of the storm vortex, and reveals that the transport mechanism that governs entrainment into and detrainment out of the typhoon is lobe dynamics. For example, the region of fluid colored brown and enclosed by the intersection of the repelling and attracting LCS is a lobe that will be detrained out of the hurricane. Indeed, without computing the LCS, the location of the boundary to the storm is not clear, and hence the concepts of detrainment and entrainment are not well-defined. Computations of the LCS for several typhoons has revealed that mixing via lobes is a dominant feature in tropical storms.

Oceanic Vortices. Elucidation of transport processes in oceans at the mesoscale is desirable for understanding local upwelling events, movement and genesis of biomass, and optimal pollution release [4]. We have computed LCS using surface velocity data provided by the Regional Ocean Modelling System at the Jet Propulsion Laboratories for a region of coast near Monterey Bay, California. The FTLE computation reaveals sharp LCS that govern the structure and transport in the flow. In Figure 2, we see that the LCS clearly demarcates an eddy at the mouth of Monterey Bay. Furthermore, the repelling surfaces (red) indicate lobes to be entrained, while the attracting surfaces (blue) indicate lobes that have been detrained. The LCS, especially when viewed as a movie in time, bears striking resemblance to the mixing process observed for Typhoon Nabi.

Medditerranean eddies are an interesting class of oceanic vortices in which dense salty water exits the Mediterranean Sea through the Straits of Gibraltar and descends into deeper water in the North Atlantic. Frictional and Coriolis effects induce rotation, resulting in a coherent ellipsoidal eddie of high salinity with a typical radius of 100km at a depth of 1km [1]. A remarkable feature of these eddies is that they retain their coherent shape and salinity and have been tracked for several years as they migrate across the Atlantic toward the Carribbean. A simple quasi-geostrophic fluid model for the flow of these eddies has been proposed by Meacham et al and shows excellent agreement with field observations [8; 7]. Our purpose in studying this model using LCS is to understand the structure of particle trajectories surrounding the ellipsoidal vortex. Figures 3(a) and 3(b) provide a side and top view of the three-dimensional LCS in the vicinity of the ellipsoid. Clearly, the background shear causes the formation of lobes and the method by which fluid is transported into the vicinity of the ellipsoid is through the mechanism of lobe dynamics. For example, fluid inside a red lobe will be entrained into the region surrounding the vortex.

Conclusion. The use of FTLE to discern LCS in flows surrounding vortices yields novel insight into the transport mechanisms at work in geophysical vortices. For both atmospheric and oceanic flows, examples have been provided in which the LCS reveals that the mechanism of lobe dynamics plays an important role in the processes of entrainment and detrainment. Furthermore, calculation of the the lobe structures allows for quantification of mixing across the boundary of the vortex.

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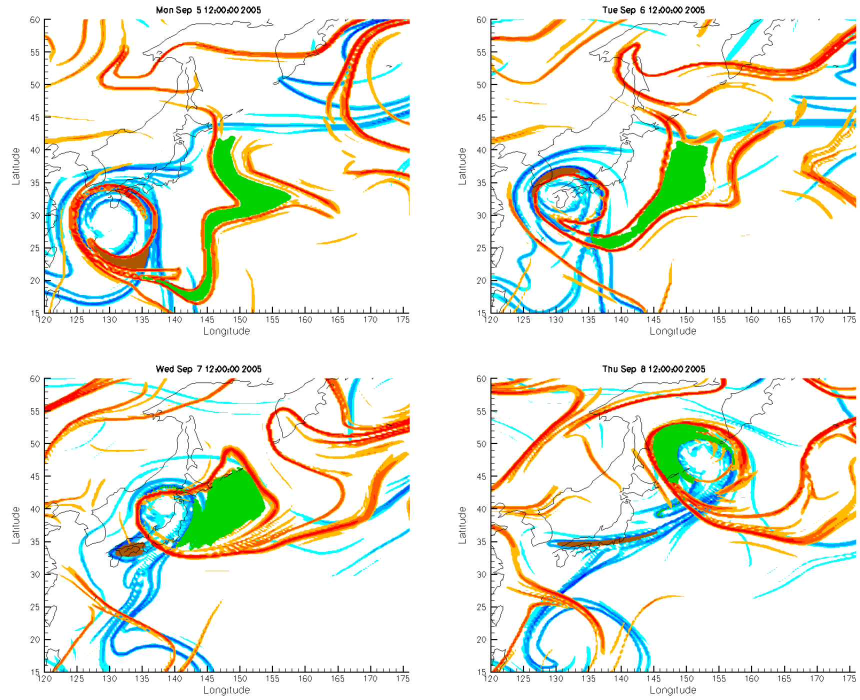


Figure 1: LCS for typhoon Nabi, computed from NCEP/NCAR Reanalysis Data at the 850mb pressure level. The intersections of the repelling (red) and attracting (blue) LCS define lobes that enclose regions of fluid that will be either entrained into or detrained out of the cyclone. For clarity, only two lobes have been colored although many more are evident during the animation. The LCS reveals that transport into and out of the cyclone is well-described by lobe dynamics. Indeed, the LCS forms a boundary to the cyclone that is exactly a homoclinic tangle from geometric dynamics theory. Over the three day period shown, the green lobe is entrained, while the brown lobe is detrained.

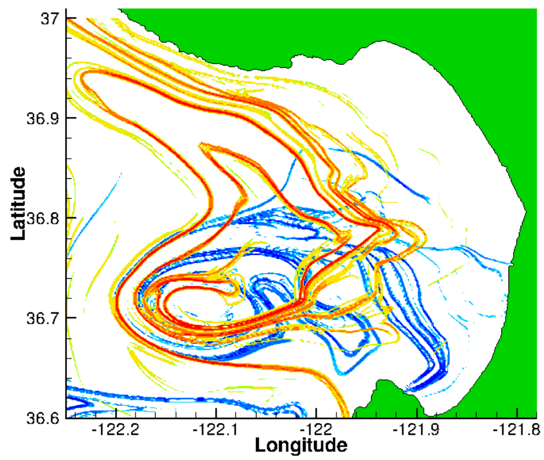
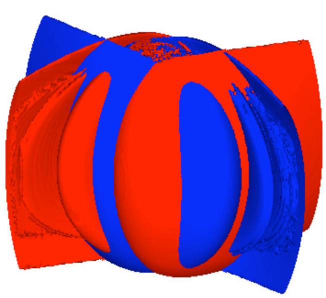
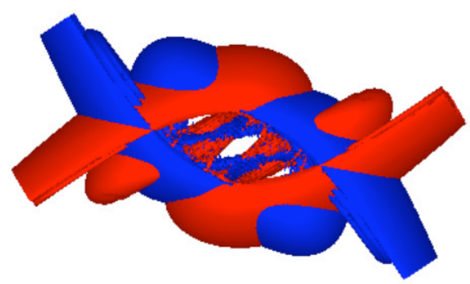


Figure 2: LCS computed using velocity data from an ocean model indicate the presence of an eddy at the mouth of Monterey Bay. The LCS demarcate the boundary of the eddy in the lower left. The repelling LCS (red) designate lobes in the upper left that will be entrained into the vortex, while the attracting LCS (blue) reveal lobes that have been detrained in the lower right.



(a) Side view of the repelling (red) and attracting (blue) LCS in the the flow surrounding the ellipsoid. The blue lobes are detrained out of the interior, while red lobes are entrained.



(b) Top view of the LCS in the flow surrounding the ellipsoid.

Figure 3: LCS in a quasi-geostrophic model for Mediterranean eddies.