NEW EXACT SOLUTIONS IN VORTEX DYNAMICS

E.Yu. Meshcheryakova

Lavrentyev Institute of Hydrodynamics, Lavrentyev prospect 15, Novosibirsk 630090, Russia; e-mail: helenmesh@gmail.com

Rotationally-symmetrical motions of ideal incompressible liquid present a great interest since they may be helpful in description of natural disasters such as whirlpools, waterspouts, tornado and cyclonic vortices. Moreover, a model of swirling motion of effectively inviscid liquid [1] is widely used in technical applications.

For plane and axial-symmetrical motions of ideal incompressible liquid there were proved global existence and uniqueness theorems, while in case of rotational symmetry there are still only local results. Therefore it is important to have a wide set of exact solutions to these equations. In this paper three classes of exact solutions are presented.

1. Solutions from first class describe time-periodic liquid motion in a semi-infinite cylinder, with a contact discontinuity surface, to one side of which the liquid is moving like a rotating piston. This motion is generated by interaction of sources and vortices distributed along the axis of symmetry of the cylinder. It was shown that by choosing the initial value data, on the axis of symmetry a source with infinitely small support can be modeled. A new class of Grad-Shafranov equation integrability was found. Also, there were obtained swirling stationary solutions describing motion in curvilinear channels, including channels with branching.

2. As it is well-known, system of Euler equations describing ideal incompressible liquid motion is of composite type. In considered reduction of these equations, their hyperbolic part was separated from hyperbolic which significantly simplified further analysis. There was considered a system of hyperbolic type with one spatial variable, which describes deformation of cylindrical layer with free surface side boundary under a stamp. In contrast to previous statements of this problem [2], the velocity field is nonlinear in terms of spatial coordinates, the motion is non-potential and there is a vortex-source on the axis of symmetry. Distinctive feature of this problem is that source intensity can be set as an arbitrary function of time and an initial boundary problem can be considered for the defining equation. A special attention was paid to investigation of contact discontinuity and jumps in vorticity, which are typical for this class of motions. In particular, it was shown that the condition of contact discontinuity absence in solution to the free boundary problem has a non-local character.

3. In contrast to gas dynamics, there are no shock waves in incompressible liquids, however, motions with contact discontinuities do exist. In ideal incompressible liquid, discontinuity of velocity and its derivatives existing at initial moment of time stay for all time of solution existence. There is a number of examples of adjunction of potential flow with a vortex flow or a swirling flow with a non-swirling flow for stationary motion of ideal liquid (for example, Hill's spherical vortex [1]). However even in case of rotational symmetry such examples are just a few. In this paper there were built two non-stationary solutions in which the domain of vortex flow is separated from the non-swirling domain by a surface of weak discontinuity in rotationally-symmetrical motion of ideal incompressible liquid. One of the solutions gives an example of motion in which velocity field derivative has discontinuity and the classical solution exists only within limited time, however, this solution was extended for all values of time as a generalized one, with velocity field being continuous.

References

[1] G.K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge Univ. Press, 1967.

[2] L.V. Ovsyannikov, "General equations and examples", in: *Problem on Unsteady Motion with a Free Boundary* [in Russian], Nauka, Novosibirsk (1967), pp.5–75.