Background current concept and chaotic advection in an oceanic vortex flow.

Koshel K.V.,[a] Stepanov D.V.,[a]

[a]Pacific oceanological institute FEB RAS, Baltiiskaya 43, Vladivostok, Russia, 690041

It is known that the fundamental dynamical relation governing the motion of a homogeneous inviscid rotating fluid is the law of potential vorticity conservation, relating in particular relative vorticity generation to the topographic irregularities of the bottom [1]. Numerous examples of quasistationary topographic vortices [6] which coincide with natural habitat of certain biological objects provide stimulus to study the interaction of these vortices with unstationary incident current. The simplest problems for models of geophisical hydrodynamics are easily derived on the basis of background current concept [7] for quasigeostrophic approximation, which makes it possible to express dynamically consistent streamfunction $\psi(x, y, t)$ in closed form (in quadratures) for any given flow region, bottom topography and boundary conditions, provided that corresponding Green function for Laplace operator is known.

The way to construct the background current is to consider a homogeneous potential vorticity which automatically satisfy to the law of potential vorticity conservation. The result will be depending from the potential vorticity value and flow can change radically for different potential vorticity values. We can define the single background current for which the potential vorticity $\bar{\Pi}$ impose a minimum of mechanical energy [7].

In this case the equations of fluid particles motion have Hamiltonian form

$$\dot{x} = u \equiv -\psi_y, \qquad \dot{y} = v \equiv \psi_x, \tag{1}$$

where $\Omega = v_x - u_y \equiv \nabla^2\psi$ is vorticity.

In general case we will have $\psi = \psi_0(x, y) + \psi_1(x, y, t)$, where the first term ψ_0 represents the planetary-topographic vortical component, and nonvortical perturbation ψ_1 is due to boundary conditions. For nonstationary flows ψ_1 it represents a wide class of models having chaotic properties [2].

Much attention was given to studies of chaotic mixing in unsteady oceanic vortex flows over irregularities in the underlying surface [3,5,8]. The generation of regular free and trapped topographic vortices as a result of interaction between an oceanic current and bed relief irregularities has been studied well [6].

In this study, we discuss the possible regular and chaotic regimes in the problem of a quasiperiodic flow around a submarine elevation of Gaussian form.

The only solution of $\nabla^2\Psi_1 = 0$ for which the velocity is bounded over a boundless plane is a spatially homogeneous flow directed at an angle θ to the x axis, with a stream-function $\Psi_1 = (B\sin\theta + v_0)x - (A\cos\theta + u_0)y$.

Here, let consider the stream-function in the form $\Psi(r,\, \varphi,\, t) = \int\limits_0^r V(\rho)d\rho - u(t)r\sin\varphi$, where $u(t) = u_0\Big[1 + \mu\cos(\omega_0 t + \varphi_0)\Big]$.

The expression for the azimuth velocity takes the form $V = -\frac{\sigma}{r}\int\limits_0^r h(\rho)\rho d\rho = -\frac{\sigma}{r}\int\limits_0^r he^{-\alpha\rho^2}\rho d\rho = \frac{\sigma}{2\alpha r}\Big(e^{-\alpha r^2} - 1\Big)$. Here $\sigma = O(1)$ is a topographic parameter.

Let us consider the case $u(t)$, $\dot{u}(t) \ll 1$, where, on the one hand, the effect of the topographic trapping is the strongest, and, on the other hand, $u(t)$ can be used as a small parameter for asymptotic analysis. Using the Zaslavsky method [9], we can obtain the estimates for the width of the nonlinera resonances over the frequency and estimate the resonances overlapping. These relationships for the overlapping criterion allow us to analyze the specific features of chaotization of the phase space for different perturbation frequencies. Figure 1 gives Poincaré cross-sections calculated for four characteristic values of the perturbation frequency.

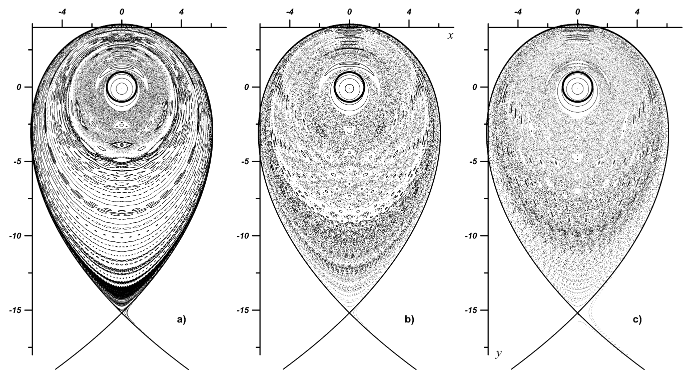

Figure 1: Poincaré cross-section at $\alpha \approx 1.256$, $u_0 = 0.092$, $\mu = 0.9$, $\varphi_0 = 0$ for the cases: a) $\omega_0 = 0.5$, b) $\omega_0 = 0.4$, c) $\omega_0 = 0.35$.

We can see the chaotic region near the separatrix and another one near center of the vortex, with regular regions between chaotic one and near elliptic point. The regular barrier can be destroyed for some frequency values. The same results give as the overlapping criterion.

The example illustrate the significant role of nonlinear resonances in chaotic advection effects induced by vortex flow [4]. Also we will consider a role of stratification in two-layered background current model.

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