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## Robustness of insect's free-flight in terms of flapping motion and vortex patterns

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## summary

Many insects flap their wings horizontally with changing the angle of attack when they stay at a particular position ("normal hovering"). This fact suggests that normal hovering is robust for the change of the detailed flapping motion and wing shape etc., which has not been clarified yet. We study this problem by a numerical model in terms of the vortex pattern generated by flapping wing and the center-of-mass stability of the insect. Numerical calculation under the influence of the gravity revealed that the stable hovering is possible by horizontal flapping regardless of the detailed flapping motion. Focusing on the flight at a critical point, it is shown that there is a transition of vortex patterns from the inverted Kármí vortex allowing a hovering to a deflected wake which does not allow hovering.

## INTRODUCTION

Insects utilize vortex generated by flapping wing, by which they achieve high performance on generating force and manouvering. Several mechanisms of efficient force generation have been proposed, but the complete understanding of the flapping flight using vortex has not been done yet. In particular, the behavior of the center of mass(CM) during flapping flight including the stability of CM (free-flight) has not been well understood, except for several experimental studies and numerical models[1, 2, 3, 4, 5]. Here we consider the robustness of the flapping flight using vortex. It is frequently observed that many insects hover by using horizontal flapping motion with changing the angle of attack(hormal hovering). Thus it is expected that the hormal hovering is robust, i.e., hovering is possible regardless of the details of the flapping motion. In this paper, we confirm this conjecture by using a two-dimenional model[1].

## THE MODEL

In Fig. 1(A), the configuration of two-dimensional horizontal flapping model is shown[1]. The center of the elliptic wing $X=(x, y)$ oscillates horizontally with changing the instantaneous angle of attack, $\alpha$. The oscillation is determined by the function

$$
\begin{equation*}
x(t)=A \cos (\omega t), \alpha(t)=\epsilon \pi \sin (\omega t)+\frac{\pi}{2}(A \text { is a constant }) \tag{1}
\end{equation*}
$$

where $\omega$ is the angular frequency. In the horizontal flapping model, all the mass is concentrated at the center of flapping (oscillation) $G$, and we assume that the motion of $G$ is restricted to the vertical direction for simplicity. The motion of the vertical position of $G, y$, is determined by the equation of motion:

$$
\begin{equation*}
M \frac{\mathrm{~d} V}{\mathrm{~d} t}=F-\xi \tag{2}
\end{equation*}
$$

where $V=\mathrm{d} y / \mathrm{d} t, F$ is the hydrodynamic force acting on the wing calcluated by the integral of the stress tensor over the surface of the wing, $M$ is the mass and $\xi$ is an external force such as the gravity along the vertical direction.
The fluid motion is calculated in the moving frame by using the CM velocity $V \equiv \mathrm{~d} y / \mathrm{d} t$. The Navier-Stokes equations in this moving frame are expressed as follows:

$$
\begin{equation*}
\frac{D \boldsymbol{u}}{D t}-\frac{\mathrm{d} V}{\mathrm{~d} t} \boldsymbol{e}_{y}=-\frac{1}{\rho} \nabla p+\nu \nabla^{2} \boldsymbol{u}, \quad \nabla \cdot \boldsymbol{u}=0 \tag{3}
\end{equation*}
$$

where $\boldsymbol{u}=(u, v)$ is the velocity field; $D / D t \equiv \partial / \partial t+(\boldsymbol{u} \cdot \nabla)$, the Lagrangian derivative; $\nabla=\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$, the nabla; $\boldsymbol{e}_{y}$, the unit vector in the y-direction; $\rho$, the fluid density; $p$, the pressure; and $\nu$, the kinematic viscosity. The second term on the left-hand side of eq. (3) represents the acceleration term due to the moving frame. Note that a uniform flow is not assumed here, so the hydrodynamic force is generated by the flapping motion alone. The no-slip boundary condition is satisfied on the wing, i.e. $\boldsymbol{u}=\boldsymbol{U}_{B}$, where $\boldsymbol{U}_{B}$ represents the velocity on the wing.
Equation (3) was integrated by using the constrained interpolation profile (CIP) method. Details of this method, validation, and verification are done in ref.[1]. Here we control two parameters $\epsilon$ and $\xi$. In the present calculation, the Reynolds number was $R e=157$, which is in the range of the values for the typical flow for many insects' flight i.e., $10^{2}-10^{4}$.

## RESULTS

Here we focus on the characteristics of the ascending flight, i.e., the flight with upward velocity. When $\xi$ is changed, the steady state of this model shows a bifurcation. In particular, when $\xi$ is increased, the period-averaged CM velocity $\langle V\rangle$ is
decreased, and at a critical value $\xi=\xi_{c}$, the steady state representing the ascending flight loses the stability. Then it shows a transition from the ascending flight to a descending flight state with negative velocity, in which the CM fluctuation is large and non-periodic[1]. Similar behavior is observed in a different model[2].
We focus on the critical velocity $V_{c}$, which means the minimum velocity in the ascending flight. Analyzing the behavior of $V_{c}$ in terms of the detailed flapping motion controlled by $\epsilon$, we discuss the characteristics of flapping flight and its robustness. A similar method [2] was used to discuss the effect of separation vortex in a symmetric flapping model[3, 4]. In Fig.1(B), a snapshot of the vorticity field for the case of $\epsilon=1 / 4$ is shown. An inverted Kármán vortex street is generated. Details of the behavior of the model at this parameter were in ref. [1]. Fig.1(C) shows a snapshot of the vorticity field for the case of $\epsilon=0$. In this case, the flapping motion is symmetric with respect to the horizontal line through $G$. The vortex pattern in this figure is, however, deflected unlike Fig.1(B). This situation corresponts to the studies in [7, 6], in which an deflected vortex street is generated.
These two types of vortex pattern define two types of flapping flight, and they are given by changing the parameter $\epsilon$ alone. However, the difference of the characteristics of each flapping flight is unclear. Here, we characterize each flapping flight by the minimum CM velocity $V_{c}$. In Fig.1(D), $V_{c}$ is shown as a function of $\epsilon$. It shows a transition: when $\epsilon<\epsilon_{c} \simeq 0.03$, $V_{c}$ take large values, which means that the model can not perform stable ascending flight with small velocity because $V>V_{c}$ for ascending flight with different $\xi$. In other words, this flapping motion does not allow hovering. Moreover, $\langle V\rangle$ shows an oscillation whose period is about 10 times larger than the flapping period. The amplitude of the long-time oscillation is indicated by error bars. On the other hand, when $\epsilon>\epsilon_{c}, V_{c}$ take relatively small values. In this region, the model can hover in a practical sense.
We calculated $\langle V\rangle$ for the parameter range $(\epsilon, \xi)$, and it is shown that the ascending velocity can be controlled by changing $\epsilon$ (data not shown). Also, in a wide range of the external force, the model can control their CM speed by changing its flapping motion $(\epsilon)$. In this range, it can hover in a practical sense. However, if the flapping motion is restricted so that $\epsilon \simeq 0$, hovering is not achieved due to an instability discussed above, even if the external force $\xi$ is small.
In summary, a simple two-dimensional model exhibits two types of flapping flight by changing the details of the flapping motion. Using a method proposed in ref.[2], we succeeded in characterizing each flapping flight mechanism using vortex. The flapping motion similar to that observed in the hormal hovering is appropriate for controlling CM velocity including hovering. Other flapping motion which causes a deflected vortex pattern does not allow the model to hover.


Figure 1. (A)Configuration of the horizontal flapping model. A single elliptical wing flaps horizontally with a change in $\alpha(t)$. The center of the flapping is assumed to be the center of mass $G$, where all the mass is concentrated. The center of mass $G$ is assumed to move in the vertical direction alone. (B) Snapshot of vortex pattern when $\epsilon=1 / 4$. (C) Snapshot of vortex pattern when $\epsilon=0$. (D) $V_{c}$ as a function of $\epsilon$. A transition is observed at $\epsilon=\epsilon_{c} \simeq 0.03$.

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