Redistribution on hierarchically refined grids for Lagrangian vortex element methods

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Lagrangian vortex element methods have the advantage that they automatically adapt to the flow field. Nevertheless, this self-adaptivity has to be controlled. The first reason arises from accuracy considerations: in order for vortex methods to converge to solutions of the Navier-Stokes equations, the particle cores should overlap. Since the flow strain causes local clustering and depletion of particles, it is necessary to control the particle distribution. Beyond this particularity of Lagrangian methods, there is another consideration that is related to both accuracy and efficiency: it should be possible to control the particle resolution depending on the local physics of the flow. Several approaches exist to maintain the spatial regularity of the particles, but the most effective one is the particle redistribution approach [2, 4]. A simple approach to achieve a varying spatial resolution is to use a mapping of the physical domain during the redistribution stage [3, 2, 4]. However, the flexibility of a global mapping function is limited and is therefore not well suited for situations where localized refinement is required.

We have developed a new redistribution approach where particles are redistributed on a locally refined cartesian grid. This approach allows for arbitrary refinement of the spatial accuracy where the flow physics require it. So far, it is not a dynamically adapting method; but some refinement criterion could be integrated to do so in the future. One important difference with the AMR method developed by Bergdorf *et al.* [1] is that there is no domain decomposition involved: at all times, a single set of particles is used. It therefore does not require any modification to the global algorithm of a classical Lagrangian vortex method.

Figure 1 shows that, in order to obtain a consistent representation of the vorticity field, new particles are located at the center of each grid cell. The new redistribution procedure can be summarized as follows: (i) Determine the set of target particles affected by the redistribution of an old particle; (ii) Estimate the "weight" of target particles using a classical high order interpolation scheme (which is non conservative when multilevel target particle are involved); (iii) Enforce conservative "weights" while minimizing difference with estimated weights. The redistribution scheme itself is consistent in the sense that it falls back to the classical high order scheme when applied to a particle being redistributed on a uniform grid. A correction procedure was successfully developed to eliminate spurious oscillations that appear at the interfaces between different levels of refinement. The issue of cross level particle interactions was addressed for the vortex method to conserve its second order accuracy.

The hierarchical redistribution procedure was used to successfully perform the simulation of the flow past a hemisphere at $Re = 3,000$ (Fig. 2). The simulated turbulent flow and the grid configuration are really challenging for the hierarchical grid refinement approach from the point of view of information transfer between grid levels. The results show that the hierarchical redistribution procedure is able to handle three-dimensional, complex, turbulent flows using relatively involved grid refinement regions.

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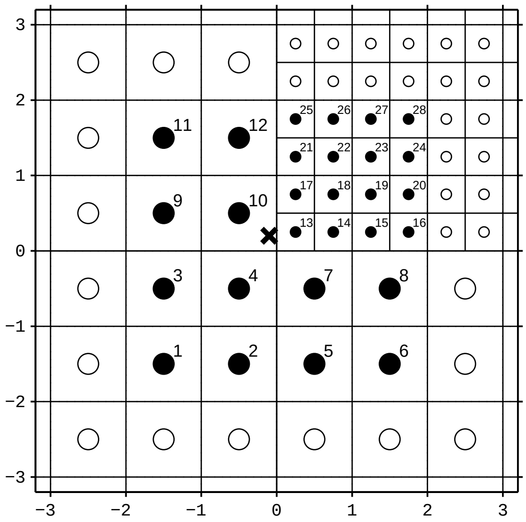

Figure 1: 2-D redistribution of a particle on a hierarchically refined grid: old particle (×), new target particles (•) and new unaffected particles (∘).

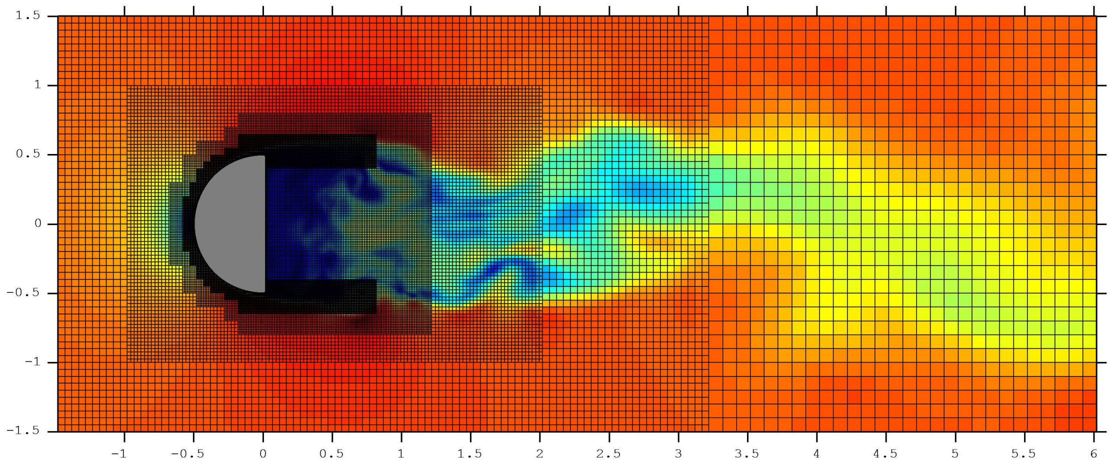

Figure 2: Hierarchical grid for the flow past the hemisphere at $Re = 3,000$: x-y slice through the 3-D grid. The field is a snapshot of the velocity norm at a time when the flow is in the turbulent regime.

References

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