Chaotic vibration of the liquid ellipsoid filled with vortical fluid

A. V. Borisov, A. A. Kilin, I. S. Mamaev

Institute of Computer Science, Izhevsk,
1, Universitetskaya str.,
Izhevsk 426034, Russia,
phone/fax: +7 (3412) 500295,
email: borisov@ics.org.ru

The dynamics of a self-attracting liquid ellipsoid is investigated. The liquid is assumed to be ideal and liquid particles attract each other with a standard (Newtonian) gravitational force. We study vibration (pulsation) of the ellipsoid assuming that the ellipsoid is allowed to rotate about one of its major axes. Riemann proved that the evolution of the semi-axes a, b and c of such an ellipsoid is governed by the equations

$$\ddot{a} = \frac{2\sigma}{a} - U_* a, \ddot{b} = \frac{2\sigma}{b} - U_* b, \ddot{c} = \frac{2\sigma}{c} - U_* c \tag{1}$$

and the volume preserving condition

$$abc = V_0 = const. \tag{2}$$

Here σ is an undetermined Lagrange multiplier. The physical meaning of σ is that it equals the pressure at the centre of the ellipsoid; $U_*(a, b, c)$ is the effective potential, which in the case of rotation about the major semi-axis c reads

$$U_* = -2\pi\varepsilon \int\limits_0^\infty \frac{d\lambda}{\sqrt{(1+\frac{\lambda}{a^2})(1+\frac{\lambda}{b^2})(1+\frac{\lambda}{c^2})}} + \frac{c_1^2}{(a-b)^2} + \frac{c_2^2}{(a-b)^2}. \tag{3}$$

Here ε is a constant proportional to the gravitation constant, c_1 and c_2 are the values of the integrals of motions (which are analogs of the angular momentum and the integral of areas).

Using the Bolin transformation, well-known in celestial mechanics,

$$x + iy = \rho e^{i\psi} = (a + ib)^2,$$

equations 1 can be shown to be Hamiltonian with the Hamiltonian

$$H = 2\rho\left(p_x^2 + \frac{y^4 p_y^2}{y^4 + \bar{c}_0^2\rho}\right) + U_*(x, y). \tag{4}$$

Here $\rho = \sqrt{x^2 + y^2}$. Obviously, the dynamical system (4) is defined in the upper half-plane $y > 0$.

Poincaré sections for the system (4) reveal chaotic regimes of vibration of the ellipsoid which is in support of Kirchhoffs conjecture that equations (1) cannot be integrated in terms of quadratures.