Vortex Ring Velocity and Minimum Separation in an Infinite Train of Vortex Rings Generated by a Fully-Pulsed Jet

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Jet pulsation engenders the formation of a vortex ring with each jet pulse leading to a train of (initially) co-axial vortex rings. The effect is most pronounced in fully-pulsed jets where a period of no-flow appears between pulses so that each pulse is similar to a starting jet. The vortex ring formation process is responsible for a number of interesting characteristics of pulsed jets including enhanced entrainment [1, 9] and thrust augmentation [4]. The role of vortex rings as dominant structures in pulsed jets has led to 2D [7] and axisymmetric [10] models of pulsed jets for aquatic propulsion based on infinite trains of equally spaced vortex pairs (2D) or rings (axisymmetric). These models, however, do not constrain the properties of the vortex train by the characteristics of the generating jet, allowing the circulation and/or spacing of the vortices to take on arbitrary values. In particular, the analysis of Weihs [10] predicts the thrust generated by a fully-pulsed jet can be enhanced by an increase in the velocity of the vortex ring train compared to that of an isolated vortex ring if the ring separation is reduced below about 3 ring radii for a given ring circulation, Γ, core radius, e, and ring radius b. Weihs notes that ring separation may be reduced by increasing the pulsing frequency, but no analysis of the minimum ring separation possible for a given Γ, e, and b is provided.

The present analysis considers the relationship between the downstream properties of a full-pulsed round jet and the pulsing parameters of the jet. Following Weihs [10], the downstream jet structure is idealized as an infinite train of thin vortex rings of ring radius b and dimensionless core radius $\varepsilon = e/b << 1$ separated by a constant distance a. Focusing on the special case of the minimum a/b achievable, the pulsed jet is idealized as a series of jet pulses of constant velocity U_0 that are adjacent in time, namely, $t_p = T$ where T is the pulse period and t_p is the pulse duration. Under these circumstances the jet may be viewed as a steady jet with periodic disturbances that break up the jet shear layer to form the vortex ring train. Then assuming inviscid behavior and following Talyor [8] and Mohseni and Gharib [6], among others, the properties of the vortex ring train may be related to the parameters of the generating jet by matching circulation, impulse, and kinetic energy for the two situations to give

$$U_0 L/2 = \Gamma \tag{1}$$

$$\rho U_0 L A = I_0 \tag{2}$$

$$\rho U_0^2 L A/2 = E_0 + E_i \tag{3}$$

where $L = U_0 t_p$ is the length of the fluid slug associated with each pulse, I_0 is the hydrodynamic impulse of each ring, and E_0 and E_i are the kinetic energy of an isolated ring and that induced by the rest of the rings in the infinite train, respectively. First order formulae for thin rings [2] are used to compute I_0 and E_0, while E_i is computed assuming the induced effect of the train on an individual ring is that of a train of point vortex rings ($\varepsilon \to 0$), which gives an integral over a convergent infinite sum that may be readily evaluated numerically. For periodic pulsing, the pulsing frequency must match the rate at which rings cross a downstream plane, giving an additional matching constraint, namely

$$T = a/W \tag{4}$$

where W is the velocity of rings in the infinite train. The ring velocity W may be decomposed in to the sum of self-induced velocity (W_0) and the velocity (W_i) induced by the rest of the rings in the train. For thin rings, W_0 may be computed from the first order formula for thin rings [2] and W_i may be expressed as a convergent infinite sum [5].

Solving equations (1) – (4) numerically provides a/b, ε, and W/U_0 as functions of L/D where D is the nozzle diameter of the generating jet. The results for a/b and W/U_0 are shown in Fig. 1 for $0.1 < L/D < 4$. In the numerical solution, the infinite sums in equations (3) and (4) were truncated to 200 terms, for which case the results in Fig. 1 are accurate to within 10^{-5}. Over the L/D range considered, $0.001 < \varepsilon < 0.5$ and ε increases monotonically with L/D. The large ε (and hence large L/D) results are only approximate because the thin ring assumption is violated, but $\varepsilon < 0.2$ for $L/D < 1.5$. Additionally, results were restricted to $L/D < 4$ because multiple rings per jet pulse may be expected for $L/D > 4$ [3].

Because the generating jet was restricted to minimum pulse separation ($t_p = T$), the results in Fig. 1 (a) represent the minimum ring separation achievable with a pulsed jet. As expected, the minimum ring separation decreases with L/D since decreasing L/D corresponds to increasing the pulsing frequency under the constraint $t_p = T$. The results for ε and equation (1) indicate that ε and $\Gamma/(U_0 D)$ are functions of L/D only, so Fig. 1(a) also indicates that an arbitrarily small ring separation is not achievable for a a given Γ and ε. This conclusion agrees qualitatively with the experimental results of Krueger and Gharib [4], although they used jet pulses with non-constant jet velocity and so were not able to achieve a/b as small as indicated in Fig 1(a). Nevetheless, very small ring separations may be achieved for sufficiently small L/D. In fact, as $L/D \to 0$, $W/W_0 \to \infty$ in agreement with Weihs' [10] results. Ring circulation also decreases, however, as $L/D \to 0$ and W/U_0 remains near the slug model value of 0.5 for all L/D as shown in Fig. 1(b). Thus, even at the minimum possible a/b the velocity of the vortex ring train is not large compared to the generating jet, preventing thrust augmentation by increased W at small a/b.

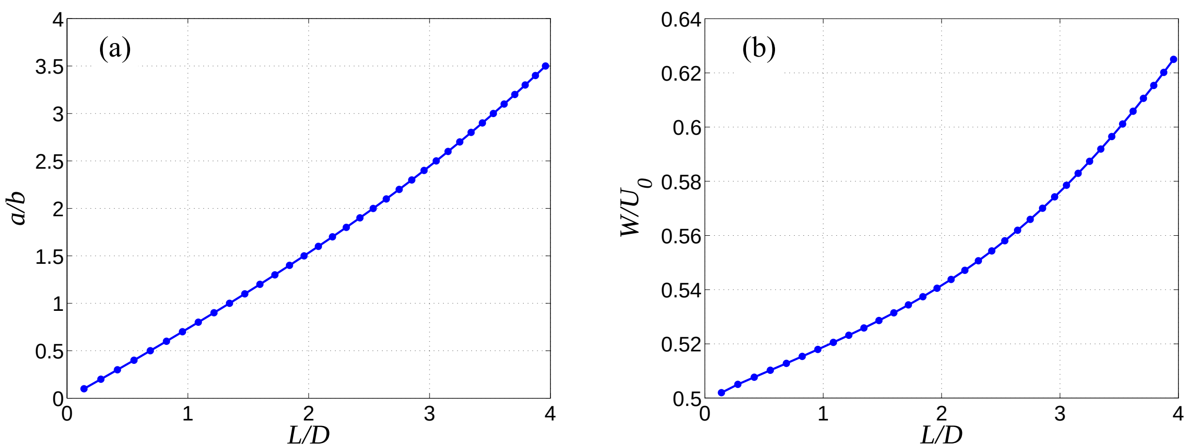

Figure 1. Relationship between properties of the vortex ring train and the stroke ratio (L/D) of jet pulses generating the vortex rings: (a) dimensionless ring separation, (b) dimensionless ring velocity.

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