Kinetic theory of two dimensional point vortices from a BBGKY-like hierarchy

Starting from the Liouville equation, I derive the exact hierarchy of equations satisfied by the reduced distribution functions of the single species point vortex gas in two dimensions. Considering an expansion of the solutions in powers of $1/N$ (where N is the number of vortices) in a proper thermodynamic limit $N \to +\infty$, and neglecting some collective effects, I derive a kinetic equation satisfied by the smooth vorticity field which is valid at order $O(1/N)$. For axisymmetric flows, this equation takes a simple form that can be studied in detail. I discuss the properties of this kinetic equation in regard to the H-theorem and the convergence (or not) towards the statistical equilibrium state (Boltzmann distribution). I also consider the relaxation of a test vortex in a bath of field vortices and obtain a Fokker-Planck equation incorporating a term of diffusion and a term of drift. The drift coefficient is related to the diffusion coefficient by an Einstein relation.