A Locally Induced Homoclinic Motion of the Vortex Filament

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An exact homoclinic solution of the Da Rios-Betchov equation is derived by using the Hirota bilinear equation. The solution describes unsteady motions of a linearly unstable spiral or wound closed filament under the localized induction approximation.

In a previous article [1] it was mentioned that the introduction of the complex-conjugate of wavenumbers in soliton solutions yields homoclinic solutions. The method leads to not only breathers or wavepacket solutions, but also homoclinic or singular solutions under specific stability conditions or dispersion relations. Subsequently, Stahlhofen and Druxes [2] claimed that some similar results were already given (see the articles cited in [2]). The author admits the insufficient references and examples in [1] in order to insist fully the usefulness of the complexification of wave numbers along with the bilinear equation for construction of N-homoclinic solutions, although the soliton-singulon solutions of the KdV equation were not found in the references of [2].

Since homoclinic solutions are considered to be more novel and its generality is still less obvious than solitons, it will be worth seeking for homoclinic solutions for other soliton equations with linear instability. In this paper, the Da Rios-Betchov equation is exemplified.

In the context of fluid dynamics, the Da Rios-Betchov equation was derived by the localized induction approximation (LIA) of the three-dimensional motion of the thin vortex filament. For its historical aspect, see [3]. The Da Rios-Betchov equation is equivalent to the focusing nonlinear Schrödinger equation (NLS) through the Hasimoto transform [5] and its N-soliton solutions were known, e.g. in [6].

On the other hand, it has been remarked that the focusing NLS has another type of solutions, i.e. homoclinic solutions [7, 8]. The solutions are homoclinic to a linearly unstable plane wave. Owing to the equivalence of the two equations, we expect homoclinic solutions of the Da Rios-Betchov equation, which are *not* appropriate to call breathers as [2] claims. The prescription is as follows: 1) Find a linearly unstable homoclinic-point solution.

2) Factor out the homoclinic-point solution (a plane wave in the following case) and derive a set of bilinear equations for the remaining denoted by the fraction g/f. 3) Solve the bilinear equations with pure imaginary wave numbers for homoclinic solutions.

The explicit solutions with animation will be presented at the conference.

References

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