The vortex model of circulation flow in sea channel

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It is well known that the subject of vortex dynamics have been initiated 150 years ago by remarkable paper [1] by H.Helmholtz. We can observe the development a vortex ideas in the period 1858-1956 in the recent review [2]. Solutions of some classical problems on vortex dynamics we can find in many books (see, for example, [3-4]). We consider the simple hydrodynamic model to predict the possible global hydrological changes in Kerch channel (between Black and Azov seas) due to local coastal changes in this region, including hydrological buildings in island Tuzla. The main goal of the report is an application of vortex methods in the modeling of hydrological processes in the considered sea region.

The fluid structure and hydrological processes in sea channels, fiords and shelf regions depends on parameters of the fluid inflow, climatic conditions and details of coastline. It is well known that the coastline changes, as a result of human activity, can result in great changes in the flow structure. The building the dike (2003) in Kerch channel from Taman peninsula is an example of technological influence on hydrological conditions in the channel (compare fig.1 and fig.2). The high velocity of the flow (0.4-0.6 km/h), small depth (4-8m) and a flat ground are main features of the hydrology in Kerch channel. Increasing the flow velocity in the fairway region in two times can result in the appearance of dangerous tendencies in hydrology of Kerch channel.

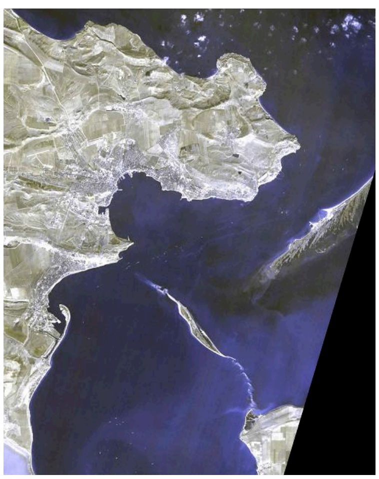


Fig.1. A satellite photography (2001) of Kerch channel between Black sea (south) and Azov sea (north)

Fig.2. A satellite photography (2003) of Kerch channel between Black sea (south) and Azov sea (north)

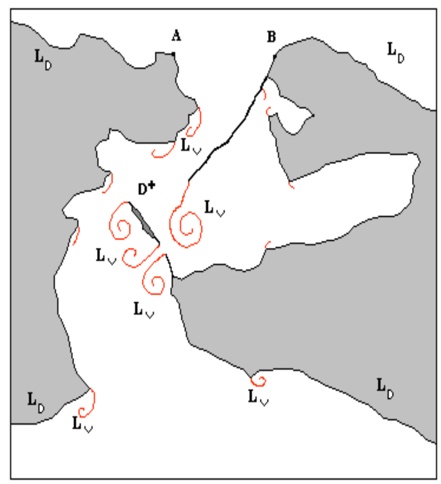


Fig.3. Scheme of hydrodynamic model of a circulation flow in sea channel

Consider the mathematical problem with multiply connected region and complex boundaries as a first approximation. The fluid flow averaged over the depth is analyzed in the region between the fixed boundaries L_d and island Tuzla (Fig.3).

Water discharge Q is a parameter of the channel. Model suppose that the fluid motion is unsteady flow, which can be described by superposition of an external potential flow due to a water discharge Q and the flow induced by vortex structures L_v forming near the sharp ledges in the boundaries.

The solution of the hydrodynamic problem reduced to the mathematical problem for complex potential $\varphi(x, y)$ in the multiply connected region D^+ with fixed impermeable L_d and moving free L_v boundaries [4-5]:

$$\Delta\varphi = 0 \text{ , at } D^+ \tag{1}$$

with boundary conditions

$$\int_A^B (\nabla\varphi, n)ds = Q \quad \text{at AB,} \tag{2}$$

$$\frac{\partial\varphi}{\partial n} = 0 \quad \text{at } L_{d}, \tag{3}$$

$$\left.\frac{\partial\varphi}{\partial n}\right|^{+} = \left.\frac{\partial\varphi}{\partial n}\right|^{-} \quad \text{at } L_V, \tag{4}$$

$$\frac{\partial\varphi^{+}}{\partial t} + \frac{\left|\nabla\varphi^{+}\right|^{2}}{2} = \frac{\partial\varphi^{-}}{\partial t} + \frac{\left|\nabla\varphi^{-}\right|^{2}}{2} \quad \text{at } L_V \tag{5}$$

and initial condition

$$L_d,\ L_O = L_v(t_O), \quad \left.\varphi^{+}\right|_{t=0} = \varphi_0^{+} \tag{6}$$

$$\text{as well as } \left|\nabla\varphi^{+}\right| < \infty\,. \tag{7}$$

We introduce the complex potential and velocity field in the following form

$$\Phi(z,t) = \varphi + i\psi = \frac{1}{2\pi i}\int_{L_d} f(\omega,t)\ln(z-\omega)d\omega + \frac{1}{2\pi i}\int_{L_v(t)} f(\omega,t)\ln(z-\omega)d\omega\ , \tag{8}$$

$$\overline{V}(z,t) = \frac{\partial\Phi(z,t)}{\partial z} = u - iv = \frac{1}{2\pi i}\int_{L_d}\frac{f(\omega,t)}{z-\omega}d\omega + \frac{1}{2\pi i}\int_{L_v(t)}\frac{f(\omega,t)}{z-\omega}d\omega \tag{9}$$

and solve the problem both for fixed L_d and for moved L_V boundaries:

For determine boundaries L_d :

$$\begin{cases} z = \omega_d(t) \in L_d,\ t > t_0: \\ \mathrm{Re}\left\{\overline{V}^{+}(\omega_d,t)\times n\right\} = 0 \\ W(t,\omega_d) = \dfrac{d\Gamma_d(t)}{dt} \\ \oint_{L_d(t)} \overline{V}(t,\xi)\,d\xi = 2C_1 \end{cases} \tag{10}$$

For vortex boundaries L_v :

$$\begin{cases} z = \omega_v(t) \in L_v,\ t > t_0: \\ \dfrac{d\overline{\omega_v}}{dt} = \dfrac{1}{2}\left(\overline{V}^{+}(\omega_v,t) + \overline{V}^{-}(\omega_v,t)\right) \\ C_1 = \displaystyle\int_{L_v} f(\omega,t)d\omega \\ t = t_0: \\ L_v(t_0) = L_{0v} \\ \overline{V}(\omega_v,t_0) = \overline{V_0}(\omega_v) \end{cases} \tag{11}$$

We propose a numerical solution of the problem (10)-(11) based on the discrete vortex method [5,6]:

$$\Phi(z,t) = \varphi + i\psi = \sum_{j_d=1}^{M_d}\frac{\Gamma_{j_d}(t)}{2\pi i}\ln(z-\omega_{j_d}) + \sum_{j_v=1}^{N_v(t)}\frac{\Gamma_{j_v}}{2\pi i}\ln(z-\omega_{j_v}(t))\,, \tag{12}$$

$$\overline{V}(z,t) = u - iv = \sum_{j_d=1}^{M_d}\frac{\Gamma_{j_d}(t)}{2\pi i(z-\omega_{j_d})} + \sum_{j_v=1}^{N_v(t)}\frac{\Gamma_{j_v}}{2\pi i(z-\omega_{j_v}(t))}\ , \tag{13}$$

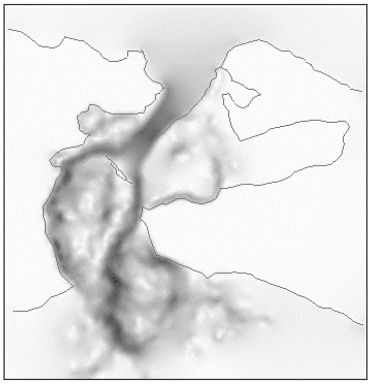
where

$$\Gamma_{j_d}(t) = \int_{L_{j_d}} f(\omega,t)d\omega\ , \quad \Gamma_{j_v} = \int_{L_{j_v}(t)} f(\omega,t)d\omega \quad j_d = \overline{1,M}_d \quad j_v = \overline{1,N}_v(t)\,. \tag{14}$$

Here unknown values $\Gamma_{j_d}(t)$, $j_d = \overline{1,M}_d$ can be found from a linear system of algebraic equations (follow from integral equation (10)) and the solution of the Cauchy's problem (11) for moving L_V boundaries.

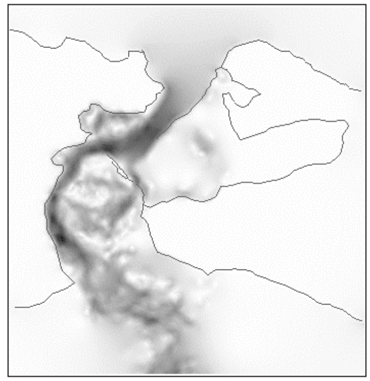
Fig. 4 and fig.5 illustrate the numeral solution of the problem (10-11). Here the dark intensity of an ink is proportional to the flow velocity in the considered channel.

The building the dike from Taman peninsula leads to an intensification of the flow both along the Crimean peninsula and around Tuzla island. Analyze show that this changes intensifies erosion processes near Tuzla and result in reducing the coastline of the island (Fig.4- Fig.5.) [6].



(a)

Fig.4. Numeral simulation of global circulation flow for the channel region (2001)

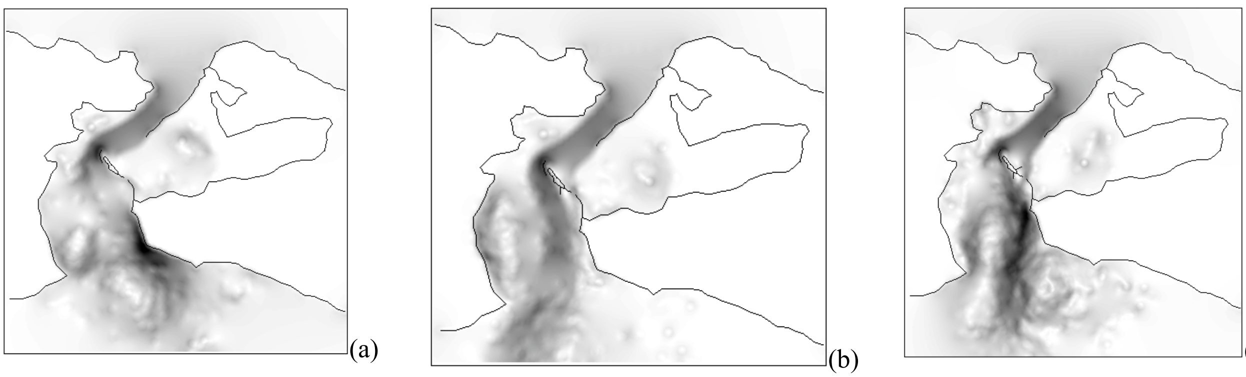


(b)

Fig.5. Numeral simulation of global circulation flow for the channel region (2003)

(c)

Fig.6. Hydrological projects for island Tuzla



(a) (b) (c)

Fig. 7. Fluid flows for different coastal buildings in according to Fig.6

Investigations show that coastal building projects (case “a”, “b”, “c” in Fig.6) on island Tuzla can change the global flow in this region (Fig.5) in comparison with the initial flow. This flow has controlled regions with local increasing of flow velocity. Analyze shows that case “b” (Fig.7) allows to displace the region with most intensive flow to the middle of Kerch channel. Therefore the problem of coastal building does not become an urgent one. Moreover, numerical simulations shows, that the flow in this case moves away impurities from channel territory and shelf zones when technocratic accidents and catastrophes are occurred.

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