## Baroclinic vortex interaction in a time-varying flow

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**Abstract** In a two-layer quasi-geostrophic model, we study the vertical interaction of two identical point-vortices in large-scale, barotropic, shear or strain flows. Vortex trajectories are determined for both steady and unsteady large-scale flows. The transition towards chaos is analyzed by means of Poincaré maps. An application to the vertical alignment of two finite-area vortices is proposed.

In stratified rotating flows, vortices can grow horizontally via merger and vertically via alignment, provided that the ambient shear, strain and vorticity gradient remain moderate (McWilliams, 1984, 1990). Vertical vortex alignment allows the barotropization of energy and enstrophy in decaying geostrophic turbulence (Rhines, 1979 ; Salmon, 1980). But when two like-signed vortices interact in a turbulent field, the shear or strain exerted by neighboring vortices on them is not stationary.
This remark prompted us to study the influence of an external shear or strain on vertical vortex alignment in a two-layer quasi-geostrophic model on the f-plane. This external flow can have both a stationary and an unsteady component. The present study generalizes previous work on 2D vortex interaction (Maze et al., 2004) to stratified rotating flows.

In a first approach, we study the motion of vortex centers and idealize the two vortices as pointlike structure. Thus we consider two identical point-vortices (one per layer) with strengths $\Gamma$, located at $r_{1,2} = \rho$, $\alpha_{1,2} = \theta$, $\theta + \pi$, embedded in barotropic, center-symmetric strain and rotation (with strain rate $S$ and rotation rate $\Omega$). The two layers have equal thickness $H/2$. Viscosity is neglected and therefore potential vorticity is conserved. The motion of each point vortex derives from a Hamiltonian and is governed by:

$$\begin{aligned} \dot{\rho} &= -s\rho \sin(2\theta) \\ \rho\dot{\theta} &= \frac{1}{\rho} - 2\gamma K_1(2\gamma\rho) + \rho\omega - s\rho\cos(2\theta) \end{aligned}$$

with $s = 8\pi S/\Gamma, \omega = 8\pi\Omega/\Gamma$. We set $s = s_0 + \epsilon s_1 \cos(\sigma t)$, $\omega = \omega_0 + \epsilon\omega_1 \cos(\sigma t)$ (with $\epsilon << 1$). The equilibrium positions are the center of the plane (vortices vertically aligned) and two center-symmetric positions (the stability of which depends on $s$ and $\omega$). These equilibrium positions are located inside regions of the plane bounded by separatrices. The central region is elliptically shaped and the side regions are curved lobes.

The stability study shows that there is a bi-frequency response to external strain and rotation ; in particular, frequency locking can lead to unbounded trajectories (unstable vortex motion). Thus, a multiple time-scale expantion of the equation of motion of the vortices around neutral equilibria is performed, when the forcing frequency is the eigenfrequency of oscillations around the equilibria.

The resulting amplitude equation is:

$$\partial_{t_2} A(t_2) = i\left[f_0^3 K^2 - \frac{3R}{2f_0} - \frac{10Q^2}{6f_0^3}\right]|A(t_2)|^2 A(t_2) + i\frac{(-1)^n \varepsilon_0 - \omega_0}{4f_0 K}\delta e^{if_2 t_2} \quad (1)$$

where $K$, $R$, $Q$ and $f_0$ are constants.

Equation (1) describes the slow-time modulation of vortex oscillations around their equilibrium positions, due to the unsteady strain and rotation. Indeed, for steady strain and external rotation, the vortex trajectories around the neutral equilibrium positions are circles. For a weak unsteady component of the external strain and rotation, the vortex trajectories are joined inward and outward spirals, located inside these circles. The periodic motion of the vortices along these spirals is demonstrated analytically. As the unsteady strain and rotation increase, the vortex trajectories exit these circles and approach the location of the separatrices of the steady problem (note that, due to the unsteady large-scale flow, these ex-separatrices have become entangled and are now chaotic regions). The entanglement of the homoclinic curves is shown via the calculation of the Melnikov function. Thus, for finite-amplitude external strain and rotation, the vortex trajectories enter these chaotic regions and become sensitive to initial conditions. The transition towards chaos is investigated by means of Poincare maps which show the destabilization of the KAM tori and of the cantori inside each lobe.

As an opening towards oceanic applications, we investigate the influence of an external shear/strain flow on the alignment of two identical *finite-area* vortices. We determine numerically the finite amplitude regimes (alignment, oscillation, divergence of the vortices) in the parameter plane of external strain and rotation, for several values of stratification. Consequences are drawn for vortex interactions in oceanic flows or in geostrophic turbulence.

**References**


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