Theoretical and Computational Fluid Dynamics manuscript No.
(will be inserted by the editor)

A. J. V. Milyus · E. Milyute · V. Milyuvene

On a question of definition of potential by the vortex motion of a liquid

Received: 22 February 2008 / Accepted: date

Abstract In his hydrodynamical investigations (1752-1754) L. Euler had found the basic system of equations of motion of an ideal non-viscous liquid in terms of a velocity $\mathbf{v} = \mathbf{v}(\mathbf{r})$ and pressure p of flowing particles in the continuum media, having some volume V, limited by a surface D [1]:

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{F} - \frac{1}{\rho}\nabla p \ , \tag{1}$$

based on the equation of continuity of fluid medium (Bernoulli's equation) and the principle of concept of stability:

$$\mathtt{div}\ \mathbf{P} = -\nabla p \ . \tag{2}$$

If in the equation (1) it is granted

$$\frac{d\mathbf{v}}{dt} = 0 \ , \tag{3}$$

that the equation (1) will be written in the form:

$$\mathbf{F} = \frac{1}{\rho}\nabla p \ , \tag{4}$$

showed the relation between the action of mass (a body) forces on the given volume V and the action of the pressure p on the internal surface D of the same given volume.

As it is seen from the vector notation of the equations (2-4), that here we notice that the interconnection of the law of stability with the *Newtonian potential* is obviously at the given local researched point A (where $A \in V$). To decide the question of the motion of the flow in the given researched point A, situated on the determined distance r from the center of the body, so that, by analogy of interconnection between the *force function* and the *Newtonian potential* $\varphi = \varphi(\mathbf{r})$, L. Euler had inserted the new analytical concept for the scalar potential of velocity $\varphi = \varphi(\mathbf{v})$, where the components of the velocity of a moving particle of a liquid v_x, v_y, v_z that are expressed in terms of this function as partial derivatives of the coordinates $v_x = \partial\varphi/\partial x,\ v_y = \partial\varphi/\partial y,\ v_z = \partial\varphi/\partial z$. By this case, L. Euler had found that the function $\varphi = \varphi(\mathbf{v})$ fits the equation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \ , \tag{5}$$

at the point A which is not situated inside the body ($A \notin V$). Notice that this basic equation of the theory of potential (5), known as the Laplace equation $\Delta\varphi = 0$ in the case ($A \notin V$), was found by L.Euler for the first time.

Developing later these ideas on study of dynamics of vector fields, H.Helmholtz in his remarkable works "*On Integrals of the Hydrodynamics Equations which express Vortex-Motion*"(1858) and "*On Discontinuous Movements of Fluids*"(1868) [2] had put the foundations of dynamics of vortices and had examinated by general investigation these forms of motion of fluids, which can lead to integrals of equations of hydrodynamics, corresponding to the *vortex motions*. If scientists had treated

A.J.V. Milyus · E. Milyute · V. Milyuvene
The Lentvaris group of LEPS, Lithuania
E-mail: litavem@inbox.ru

before him absolutely only such problems of hydrodynamics, in which the potential of velocities exists (S.A.Chaplygin, 1902) [3], so H.Helmholtz had investigated the vector fields, where the potential of velocities has not take place. Furthermore, H.Helmholtz had laid the mathematical basis of forming of streams, producing integrals which are necessary for investigation of dynamics of vortex motion of fluid, proposing that "by vortex motions of fluid there are same as by electromagnetic actions, the velocities and forces outside space occupied by vortex filaments or electric currents, that depend on multivalued potential functions satisfied on a general differential equation for magnetic potential functions; by the way inside space, occupied by vortex filaments or currents, the *other functions* defined by equations" [2] describing the internal rotating motions of fluids, that appear here in place of potential functions which are not applied to this case.

Independently from theoretical investigations of H.Helmholtz, in these years J.Cl.Makswell had elaborated the united theory of Electricity and Magnetism for description of vortex electromagnetic fields, based on the experiments of M. Faraday and having generalized exciting in this time separately theories on Electricity, Magnetism and Hydrodynamics [4-7]. But, the ideas of Ampère's molecular currents had a strong influence at J.Cl.Maxwell, then, that did not let J.Cl.Maxwell definite the role of magnetic "tubes" forming the field.

The detailed conducted by us analysis of physical and mathematical concepts of dynamics of vortices, proposed by H.Helmholtz and J.Cl.Maxwell, that showed that the some facts flowing from the nature of vortex itself did not come into the view of dynamics of vortices elaborated in this time, i.e. they have remained be unnoticed [8-10]. In the given work a number of problems of a vortex motion of fluid having constant density ρ is considered.

Keywords Vortex · Potential · Vortex Motion · Electromagnetism

PACS 47.32.-y · 47.32.C-

References

1. Euler, L.: General laws of motion of liquides // Mechanica zhidkosti i gaza, Nr.6, 1999, p. 26-54 (The Russian translation by Z.N.Dobrovolskaya from the French memory of L.Euler "Principes généraux du movement des fluids "(Mémories de l'Académe royale des sciences et belles lettres, Berlin, 1757. T. 11 (1755) . P. 274-315 = Opera omnia, ser. II. V. 12. P. 54-91).
2. Helmholtz, H.: The Foundations of Vortex Theory, Moscow, Izhevsk (2002)
3. Chaplygin, S.A.: An Application to the book of Helmholtz.: In book: Helmholtz, H.: The Foundations of Vortex Theory, Moscow, Izhevsk (2002)
4. Maxwell, J.Cl.: On Faraday's Lines of Force. //Transactions of the Cambridge Philosophical Society, X, Part. I p. TBD, 1855-1856 (read Dec. 10 1855, and Feb.11, 1856)
5. Maxwell, J.Cl.: On Physical Lines of Force (1861)
6. Maxwell, J.Cl.: A Dynamical Theory of the Electromagnetic Field (1865)
7. Maxwell, J.Cl.: A Treatise on Electricity and Magnetism. Clarendon Press, Oxford (1873)
8. Miliute, E., Miliuviene, V.: The Dynamics of Substance in a Spherical Vortex and Fields of Interaction. In: International EUROMECH 448 Conference "Vortex Dynamics and Fields Interactions", Paris, France, September 6 - 10, Theses of Reports, pp. 104 - 106, École Supérieure de Physique et de Chimie Industrielles (ESPCI) (2004)
9. Milyute, E. A., Milyuvene, V. A., Milyus, A.J.V.: The Nature of Charge, the Mechanisms of Radiation and the Internal Morphology of Interacting Objects. Mokslo Aidai, Vilnius (2005) (in Russian)
10. Milyute, E., Milyuvene, V., Milyus, A.J.V.: Some Questions of Dynamics of Substance in the Spherical Vortex. In: IUTAM Symposium on "Hamiltonian Dynamics, Vortex Structures, Turbulence", Moscow, Russia, August 25-30, pp. 116-117. Institute of Computer Science, Izhevsk (2006)