Applications of 2D helical vortex dynamics

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The main applications of 2D dynamics of helical vortices embedded in flows with helical symmetry are addressed.

In contrast to the well-known Beltrami helical flows with colinear velocity and vorticity fields the class of 2D flows introduced in [1] is characterized by a linear correlation between axial and azimuthal velocity components

$$w_z + r\,w_\theta / l = const\,, \qquad (1)$$

here $2\pi l$ is the pitch of the helical symmetry and r is the radial distance from the symmetry axis.

The fundamentals of the 2D helical vortex dynamics lie in

(i) 2D algebraical expression of Biot-Savart law for helical filament [1, 2] or tube [3];

(ii) Final analytical solution for self-induced motion of the helical vortex cores [2];

(iii) Goldstein's solution for circulation of equilibrium of helical vortex sheets [4];

(iv) Generalization of Kelvin's problem on point vortex N-gon stability to helix [2, 5] etc.

The primary assumption of the 2D theory is the hypothesis of helical symmetry that has been carefully tested in swirling flows in different kinds of swirlers and vortex generators. The correlation (1) has been experimentally confirmed for a wide range of operating conditions in vortex devises; in wake behind rotors and in flows downstream wall vortex generators. Both right- and left-handed helical symmetries were found in these real flows and hypothesis of existence of possible transitions between the different types of the vortex structures have been put forward [6]. The change in axial velocity distribution from a jet-like profile to a wake-like during vortex breakdown has been investigated from this point of view and the associated transition from right- to left-handed helical symmetry of the vorticity field has been confirmed from experimental data and numerical simulations.

Included among the main applications of 2D dynamics of helical vortices are considered a number of problems on mass and heat transfer enhancement of vortex devices and vortex theory of rotors (propellers, wind turbines etc):

A) Two major factors of the mass and heat transfer enhancement in swirling pipe flows have been identified by the theoretical model: (i) formation of the swirl flow with left-handed helical vortex because right-handed vortex structures generate jet-like swirl flows and can diminish transfer; (ii) modification of the near wall velocity profile by different vorticity distribution in the vortex core.

B) Various vortex models for far wakes behind rotors are analyzed, resulting in the development of a new analytical wake model. The classical vortex model of Joukowski has been extended by specifying an inner vortex structure. The new model enables us to calculate induced velocities in very good agreement with measurements. The model may further explain the very high peaks appearing in the time signal of the axial velocity measured near the centre of the tip vortices of unsteady far wakes.

C) An analytical method to determine the loading of an optimum wind turbine rotor has been developed. The method, which basically is a modification to the original model of Goldstein, is based on an analytical solution to the wake vortex problem that enables to determine the optimum circulation distribution at all operating conditions. In contrast to earlier models by Goldstein [4] and Theodorsen [7], the new model is consistent with the well-known 'Lanchester-Betz-Joukowsky limit' and the general momentum theory and enables for the first time to determine the theoretical maximum efficiency of rotors with an arbitrary number of blades.

References

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