DYNAMICS OF A FLUID INSIDE A PRECESSING CYLINDER

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Summary This paper addresses experimentally and theoretically the stability of a fluid inside a precessing cylinder. For small Reynolds number this flow is stable and can be described as a superposition of Kelvin modes forced by the precessional motion. For larger Reynolds number it becomes unstable. A mechanism of triadic resonance between Kelvin modes explains this phenomenon and allows to determine the stability threshold, in excellent agreement with experimental results.

INTRODUCTION

The knowledge of the flow forced by a precessional motion is of critical importance in several domains (particularly in aeronautics and geophysics).

Experiments such as the one carried out by McEwan [1] clearly show that a fluid forced by precession creates a flow which can be decomposed as a sum of inertial waves (also called Kelvin modes). For low Reynolds numbers this flow is stable. For large Reynolds numbers it becomes unstable and can degenerate into a turbulent flow. To explain this instability a mechanism of triadic resonance between Kelvin modes is proposed, that bears similarity with the elliptic instability [6], [7].

PRESENTATION OF THE PROBLEM

We consider the flow inside a precessing cylinder of height H and radius R, full of fluid of kinematic viscosity ν. This cylinder rotates at the angular frequency Ω_1 around its axis. It is mounted on a platform which rotates at the angular frequency Ω_2 as shown on figure (1a). The angle between the two axes of rotation is the precession angle θ. The dynamics of this precessing system depends on four dimensionless numbers: the aspect ratio $h = H/R$, the frequency ratio $\omega = \Omega_1/(\Omega_1 + \Omega_2 \cos(\theta))$, the Rossby number $Ro = \Omega_2 \sin(\theta)/(\Omega_1 + \Omega_2 \cos(\theta))$ and the Reynolds number $Re = (\Omega_1 + \Omega_2 \cos(\theta))R^2/\nu$. An experimental set–up which allows Particle Image Velocimetry (PIV) measurements in transverse section of the cylinder has been built at the laboratory. It is described in detail in [3].

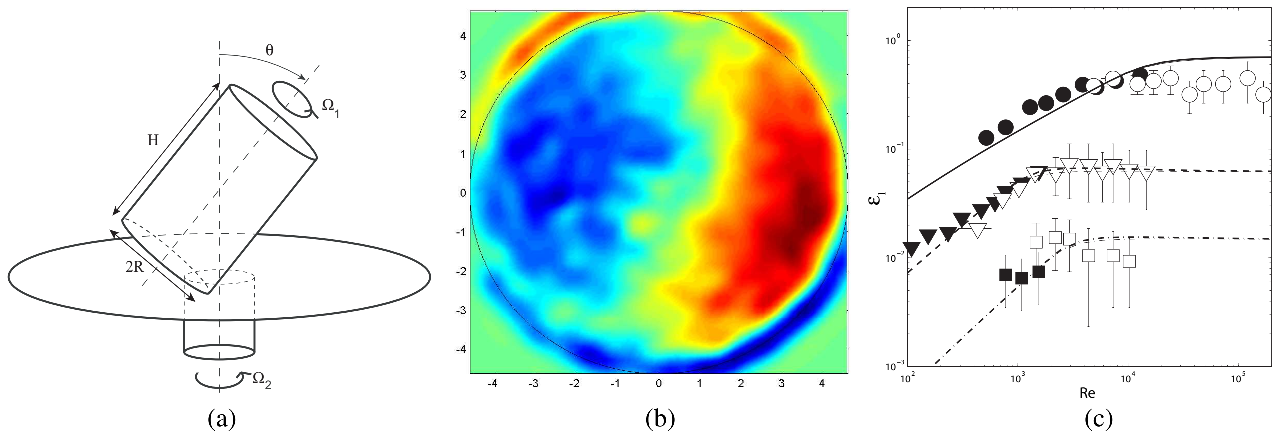

Figure 1. (a) Sketch of a precessing cylinder. (b) Vorticity field of the first Kelvin mode measured by PIV at its first resonance, in the absence of instability ($h = 1.62$, $\omega = 1.18$, $Re \approx 3500$, $Ro = 0.0031$). (c) Amplitude of the first Kelvin mode obtained at the first (solid line), second (dashed line) and third (dotted line) resonance. Symbols are experimental results ($h = 1.8$, $\theta = 2°$.)

BASE FLOW

Figure (1b) represents the axial flow vorticity, in the cylinder reference frame, for small Reynolds and Rossby numbers. The structure observed corresponds to a Kelvin mode which is excited by precession [1]. A linear and inviscid theory is sufficient to predict its amplitude ε_1. When this mode is resonant (i.e $h = n\lambda/2$, with n an odd number) ε_1 diverges. A viscous [2] and weakly nonlinear [3] theory is then necessary to predict the amplitude saturation. Figure (1c) represents the amplitude saturation of this Kelvin mode as a function of Re at its three first resonances. We show [3] that for small Reynolds numbers ε_1 scales as $Ro\sqrt{Re}$. For large Reynolds numbers, ε_1 scales as $Ro^{1/3}$. These two scalings are in good agreement with PIV measurements represented by symbols on figure (1c).

INSTABILITY

It is well known that the flow becomes unstable when the Reynolds or the Rossby number is increased [4], [5]. We show that this instability results from a triadic resonance between the forced Kelvin mode and two free modes. These free modes have been observed by PIV measurements as shown on figure 2(a,b) by looking at the vorticity in two different sections. A linear stability analysis has been developed. It leads to a formal expression of the growth rate. We show that for $h = 1.62$ and $\omega = 1.18$ the coupling between free Kelvin modes with azimuthal wavenumbers equal to 5 and 6 are the most unstable, as observed in experimental visualizations.
Including viscous effets we also determine the Rossby number for which the flow becomes unstable. This critical Rossby scales as $Re^{-3/2}$ for low Re and as Re^{-1} for high Re. These two scalings are well confirmed by PIV measurements.

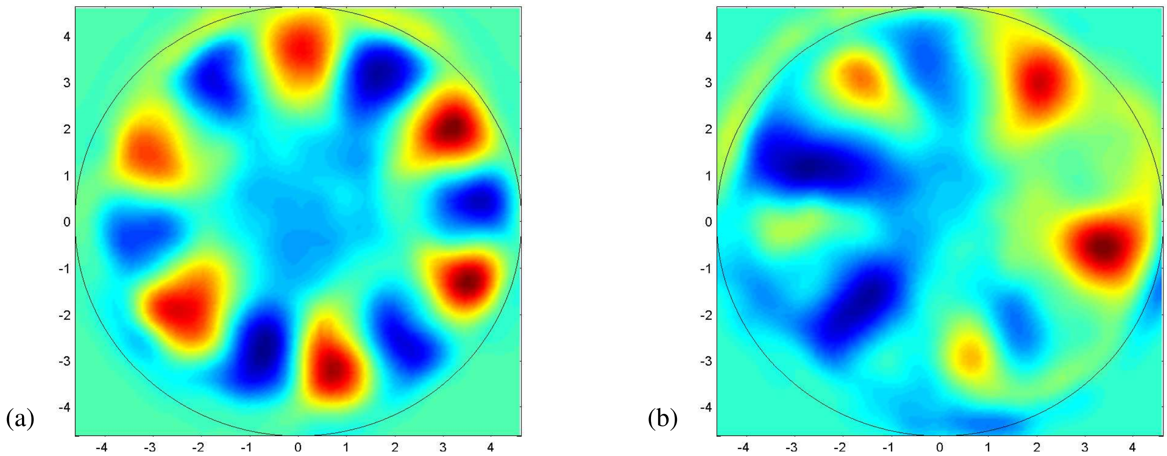

Figure 2. Vorticity field of the free modes which constitute the instability of a precessing cylinder. (a) Vorticity field measured at mid–height of the cylinder. (b) Vorticity field measured at $z = h/4$ showing the mode $m = 5$ superimposed with the forced Kelvin mode of figure (1b) ($h = 1.62$, $\omega = 1.18$, $Re \approx 6000$, $Ro = 0.0031$).

CONCLUSION

Particle Image Velocimetry measurements have shown that the flow inside a precessing cylinder is stable for low Reynolds numbers and unstable for large Reynolds numbers. In the general case, the stable flow can be predicted by a linear inviscid model and is shown to be a superposition of Kelvin modes. In the particular case of a resonant flow, we have developed a viscous and weakly nonlinear theory to predict the amplitude saturation of the resonant Kelvin mode. This theory has been confirmed by experimental results. For large Reynolds numbers, the structure of the unstable flow has been measured with PIV. We have shown that the instability results from a mechanism of triadic resonance between the forced Kelvin mode and two free modes. Moreover we have determined analytically the stability threshold, in good agreement with experiments.

References

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