Explosive instability of geostrophic vortices

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Abstract In a quasi-geostrophic model, we study the baroclinic and parametric instability of a baroclinic vortex. The singular unstable modes for potential vorticity anomalies are compared with the classical normal modes. Short time singular modes are explosively unstable and depend only on the baroclinic component of the flow. As time progresses, they evolve towards the normal modes. Parametric instability of the vortex, resulting from an oscillatory buoyancy forcing at the surface, also illustrates the importance of singular modes.

In stratified rotating turbulence and in planetary fluids, baroclinic processes have been recognized as essential in the generation and evolution of vortices. During two decades, the stability of circular geostrophic vortices has been studied with normal-mode perturbations (Pedlosky, 1985; Kozlov et al., 1986; Flierl, 1988; Helfrich and Send, 1988; Sokolovskiy, 1988; Carton et al., 1989; Carton and McWilliams, 1989; Carton and Correard, 1999); their nonlinear evolution formed more complex vortex structures (dipoles, tripoles...).

More recently, in an atmospheric context, work has been devoted to the explosive growth of a certain type of perturbations : singular modes, which are a linear combination of normal modes (Farrell and Ioannou, 1996a,b, 1999). Singular modes are the (temporarily) fastest growing perturbations of the linearized dynamical equations, with respect to a quadratic norm. In the present study, we investigate how such modes can grow on a baroclinic geostrophic vortex, in a two-layer model.

A baroclinic vortex, composed of two superimposed disks of constant potential vorticity (hereafter PV), is the mean flow under consideration (Flierl, 1988). The linearized dynamical equations, expressed in barotropic and baroclinic deviations of the potential vorticity fronts η_t, η_c, are

$$\partial_t \eta_t = -il[U_t(1-1/l)\ \eta_t + U_c(1-1/(2lI_1(\gamma)K_1(\gamma))\ \eta_c]$$

$$\partial_t \eta_c = -il[U_c(1-(I_l(\gamma)K_l(\gamma)/(I_1(\gamma)K_1(\gamma))\ \eta_t +$$

$$U_t(1-2I_l(\gamma K_l(\gamma))\eta_c + \xi U_c(1-(I_l(\gamma)K_l(\gamma)/(I_1(\gamma)K_1(\gamma))\ \eta_c]$$

where l is the angular mode, U_t, U_c the barotropic, baroclinic mean flow, $\gamma = 1/R_d$ and ξ is related to the ratio of layer thicknesses. This problem can be set in vector form as $\partial_t X = AX$. Normal modes are obtained by setting $\partial_t X = \sigma X$ where σ contains the growth rate and the phase speed.

Singular modes are the directions of fastest growth of η; note that, Q being the mean potential vorticity, ηQ is a potential vorticity anomaly, and therefore singular modes correspond to the fastest growth of the enstrophy perturbation. Singular modes are computed as the eigenvectors of $\exp(A^* t)\exp(At)$ where A^* is the transposed conjugate of A.

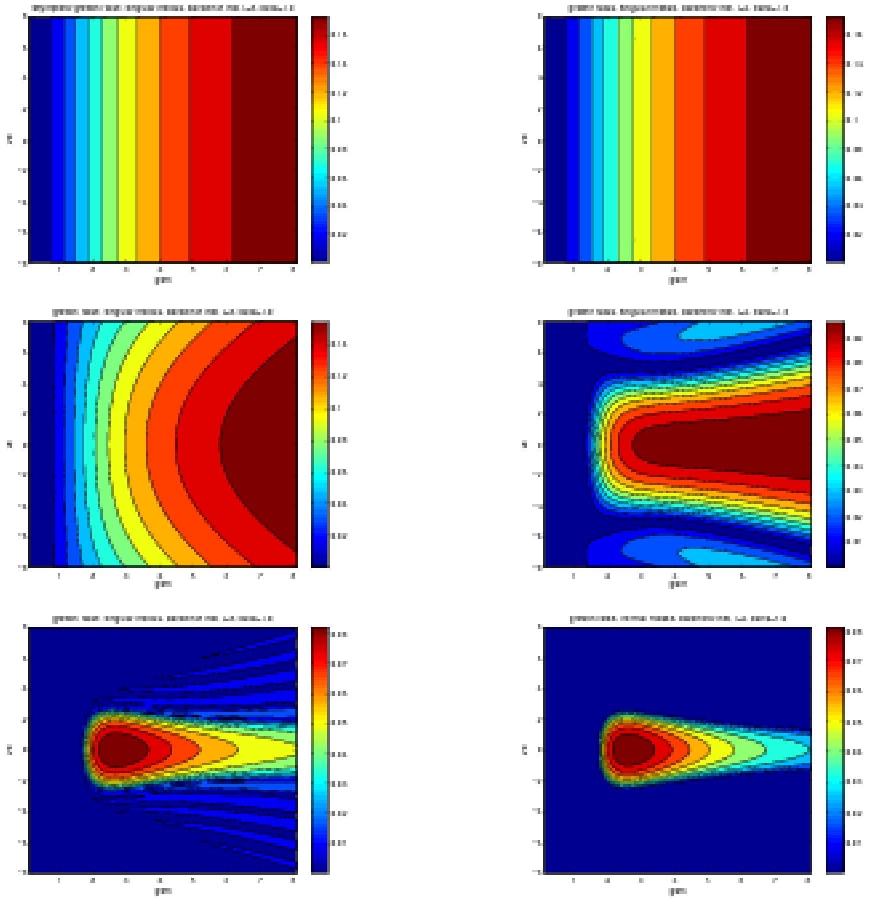

Figure 1: Growth rates of singular modes at asymptotically short time (upper left panel), at short time ($t = 0.1$, upper right), with growing time ($t = 5, 20, 100$, middle left, right, lower left) and finally those of the normal modes (lower right), in the (γ, V_0) plane (case of equal layer depths).

Figure 1 shows the growth rates of singular modes at vanishing time (asymptotic formula), at short time, with growing time and finally those of the normal modes, in the (γ, V_0) plane (with $V_0 = U_t/U_c$). Contrary to normal modes, singular modes at short time are independent of the barotropic component of the mean flow and are much more unstable.
Then we assume that a surface buoyancy forcing is applied periodically in time. This induces a periodic variation of the density interface elevation in the vortex and therefore of the baroclinic mean PV. If the pulsation of the forcing is equal to the difference of the pulsations of two neutral waves on the PV fronts, resonance can occur and parametric instability can grow. Figure 2 shows the growth rates of parametric instability in this case, near and away from marginality of baroclinic instability. This latter case shows the vertically banded structure of short-term singular modes. This similarity may be explained by the short time (half a forcing period) allowed for the growth of parametric modes which must

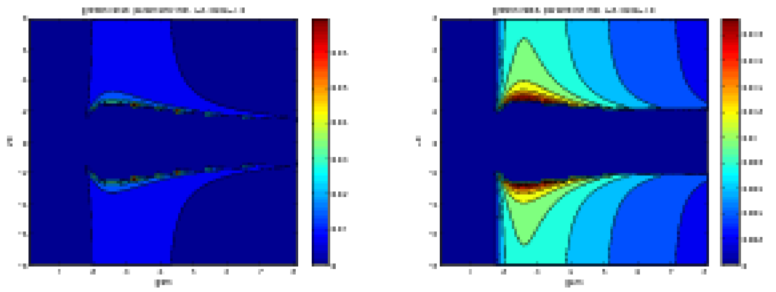

Figure 2: Growth rates of singular modes for parametric instability near marginality of baroclinic instability (left), away from marginality via filtering (right), in the (γ, V_0) plane

adopt the fastest-growing structure.

An extension to nonlinear dynamics will also be proposed.

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