Structure of a Bathtub Vortex: Importance of the Bottom Boundary Layer

Hiroshi Niino[*1], Shinji Yukimoto[*1], Takashi Noguchi[*2] and Ryuji Kimura[*3]
[*1] Ocean Research Institute, The University of Tokyo, Tokyo 164-8639, JAPAN
[*2] Graduate School of Engineering, Kyoto University, Kyoto 606-8501, JAPAN.
[*3] The Open University of Japan, Chiba 261-8586, JAPAN

Abstract

A bathtub vortex in a cylindrical tank rotating at a constant angular velocity Ω is studied by a laboratory experiment, a numerical experiment and a boundary layer theory. The laboratory and numerical experiments show that two regimes of vortices in the steady-state can occur depending on Ω and the volume flux Q: When Q is large and Ω is small, a potential vortex in which angular momentum is constant outside the vortex core is formed (Regime I). When Q is small or Ω is large, however, a vortex in which the angular momentum decreases with decreasing radius is generated (Regime II). The boundary layer theory shows that the vortex regimes strongly depend on the theoretical radial flux through the bottom boundary layer under a potential vortex: When the ratio of Q to the theoretical boundary layer radial flux Q_b (scaled by $2\pi R^2(\nu\Omega)^{1/2}$) at the outer rim of the vortex core is larger than a critical value (of order 1), the radial flow in the interior exists at all radii and Regime I is realized. When the ratio is less than the critical value, the radial flow in the interior nearly vanishes inside a critical radius and almost all of the radial flux occurs only in the boundary layer, resulting in Regime II in which the angular momentum is not constant with radius. This criterion is found to explain the results of the laboratory and numerical experiments very well.

1. Introduction

Strong natural vortices such as tornadoes and dustdevils are often modeled by a Rankine vortex (Rankine, 1882) in which the angular momentum is constant outside the vortex core. Recent observations of tornadoes using a portable Doppler radar, however, show that the angular momentum outside the core of tornadoes decreases with decreasing radius (Wurman and Gill, 2000).

In order to clarify the mechanism by which the velocity distribution of a strong vortex is determined, we have performed laboratory and numerical experiments on a bathtub vortex in a rotating tank. There have been a number of studies on bathtub vortices (e.g., Lewellen, 1962; Turner, 1966; Lundgren, 1985; Echavez and McCann,2002; Andersen et al., 2006). However, the mechanism that determines the velocity distribution of a bathtub vortex outside the vortex core has not been clarified and the effects of the bottom boundary layer on the vortex structure have not been properly considered.

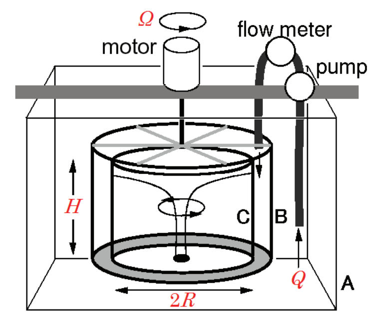

Fig. 1: A schematic of the experimental apparatus.

2. Laboratory experiment

The schematic of the experimental setting is shown in Fig. 1. The bathtub vortex is generated in a cylindrical tank of 40cm diameter which rotates at a constant angular velocity of Ω. The working fluid, fresh water, is drained through a circular hole of 2.5cm diameter at the bottom at a volume rate of Q, and the same amount of water is returned to the tank through its sidewall the upper part of which is made of sponge. The mean water depth of the tank is 18cm.

Figure 2 shows that two regimes of vortices are realized in the experiment: When Q is large and Ω is small, the angular momentum in the steady state is constant outside the vortex core (Regime I); When Q is small or Ω is large, on the other hand, it decreases with decreasing radius (Regime II).

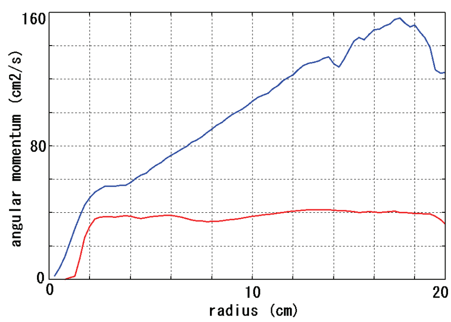

Fig. 2 Radial profiles of the angular momentum for Ω = 0.1 rad/s and Q = 100 cm^3/s (red line), and Ω = 0.4 rad/s and Q = 33 cm^3/s (blue line).

For Regime II, a flow visualization in the radial-height plane shows that a dye introduced near the top of the sidewall travels along the sidewall, goes into the boundary layer, moves radially inward and goes out of the drain hole.

3. Numerical Experiment

An axisymmetric numerical model in the cylindrical coordinate is used to predict the time evolution of the angular momentum and the streamfunction in the radial-height (r-z) plane. The boundary conditions are stress free at the central axis, free slip at the upper lid and no slip at the bottom and sidewall which are rotating at the angular velocity Ω.

Figures 3a and 3b show the streamlines in the r-z plane and the radial profile of angular momentum at the height of 10 cm from the bottom, respectively, for Regime I, and Figs. 3c and 3d those for Regime II. For Regime I, a considerable part of streamlines that start from the sidewall reaches the outer rim of the vortex core through the interior, explaining that the angular momentum becomes constant with radius (Fig. 3b). For Regime II, however, all the streamlines go into the bottom boundary layer and no radial flow exists in the interior.

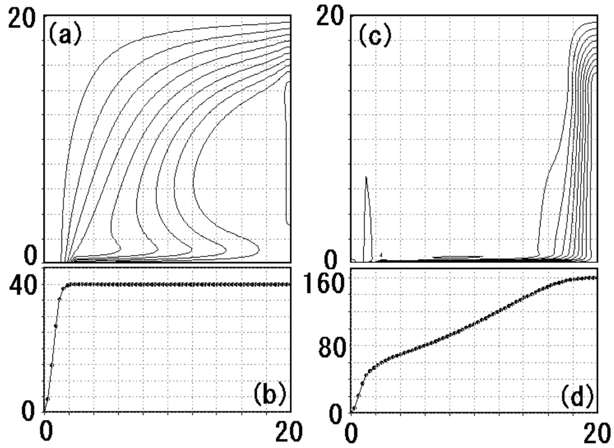

Fig. 3: The streamlines in the r-z plane (a and c) and radial profiles of angular momentum at the mid-depth (b and d). For (a) and (b), Ω = 0.1 rad/s and Q = 100 cm^3/s, and for (c) and (d), Ω = 0.4 rad/s and Q = 33 cm^3/s.

4. Boundary layer theory

Since the numerical experiment has shown that the structure of the vortex is governed by the bottom boundary layer, the characteristics of a bottom boundary layer under a potential vortex is studied by a boundary layer theory. Axisymmetric boundary layer equations are integrated with time to obtain a steady-state solution, where the side and bottom boundaries are rotating at the angular velocity Ω, the inner boundary is open, and the horizontal velocity components approach those of the potential vortex at the upper boundary. The boundary layer structure thus obtained turned out to coincide well with the one obtained for Regime I in the numerical experiment. The radial flux through the bottom boundary layer is found to be scaled $2\pi R^2(\nu\Omega)^{1/2}$ and increases monotonously with decreasing radius, where R is the inner radius of the tank and ν the kinematic viscosity.

In the present study the maximum radial flux Q_{bmax} through the bottom boundary layer is attained at the radius of the drain hole, and is given by $Q_{\mathrm{bmax}} = 0.88 \times 2\pi R^2(\nu\Omega)^{1/2}$. Thus, the criterion for the realization of a potential vortex is given by

$$Q > 0.88 \times 2\pi R^2(\nu\Omega)^{1/2}, \qquad (1)$$

which nicely explains the results of the laboratory and numerical experiments (e.g., Fig. 4). It can be further shown that a criterion for a realization of a potential vortex for the case in which the bottom of the tank is not rotating is given by a similar equation to Eq. (1) except that the proportionality constant is about 30 percent larger.

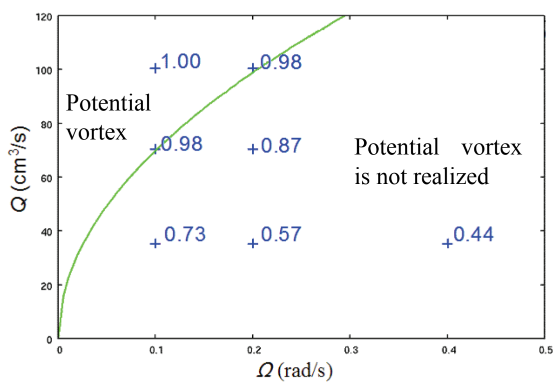

Fig. 4: The angular momentum at r=4cm scaled by that at the sidewall on the Ω-Q plane as obtained from the numerical experiment. The green curve shows the criterion given by (1).

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