Vortex structures in turbulence growth

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The statistical and structural properties of fully developed, isotropic turbulence are relatively well understood to date. Some features, such as the occurrence of a steady energy transfer in wavenumber space according to a $k^{-5/3}$ have been very well assessed both in laboratory and numerical experiments. There is also general consensus that intermittency in fully developed turbulence is due to the presence of a complex tangle or long-lived (coherent) vortical structures having either sheet-like or tube-like shape. However, much less is known about the mechanisms of formation of fully developed turbulence starting from arbitrary initial conditions, and in particular about the physical phenomena underlying the onset of an inertial range scaling. This is perhaps due to the fact that the events leading to transition to turbulence may depend upon the initialization details, and therefore are not expected to be universal.

The present work systematically investigates initial-value problems for the viscous Navier-Stokes equations, with three different initial conditions, corresponding to colliding Lamb dipoles, the Kida-Pelz and the Taylor-Green flows. Several Reynolds numbers are also considered, $Re = 1000, 3000, 5000$. The main objective of the study is to investigate the possible occurrence of a universal mechanism of transition to turbulence. For this purpose we analyze the statistical properties of the flow fields, and in particular consider the enstrophy dynamics and its relation to the energy spectra. The analysis shows that a convenient representation to search for a universal behavior is the one reported in Fig. 1, where the time derivative of the enstrophy ($\dot{\Omega}$) is plotted versus Ω.

The results indeed show some degree of universality, and all flows under consideration obey the following dynamics: i) during a first stage, enstrophy increases at a super-exponential rate due to vortex stretching ($P_\Omega = \langle \omega_i \omega_j S_{ij} \rangle$), and dissipation effects are minimal; ii) enstrophy dissipation becomes comparable to production starting from the time when $\Omega/\Omega_0 \sim 5$. At this time, which roughly corresponds to the (possible) inviscid enstrophy blow-up, very small scales are created, and both skewness and maximum vorticity attain a peak; iii) in a second stage the enstrophy continues to grow at a slower (exponential rate), with viscous dissipation ($\mathcal{D}_\Omega = -\nu \left\langle (\partial \omega_i / \partial x_j)^2 \right\rangle$) approximately balancing stretching. During this phase the energy spectrum in the high wavenumber

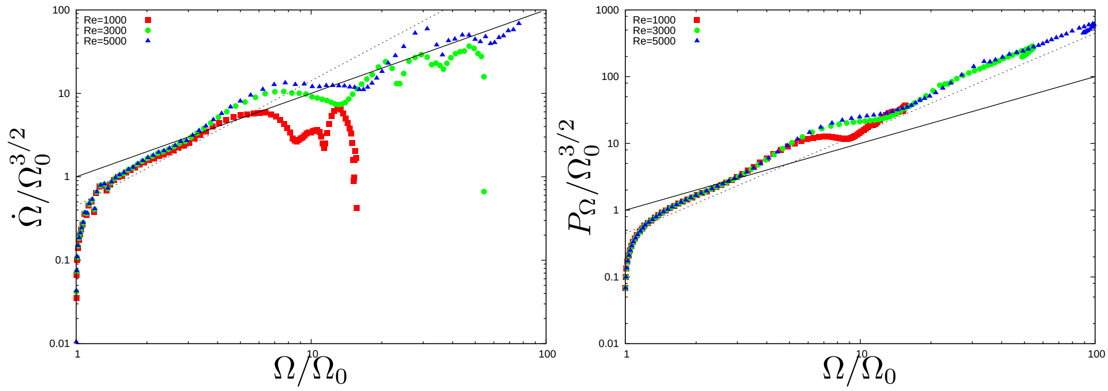

FIG. 1: Map of enstrophy amplification and production for Taylor-Green flow. Solid lines indicates $\dot{\Omega} \sim \Omega$; dashed line indicates $\dot{\Omega} \sim \Omega^{1.5}$.

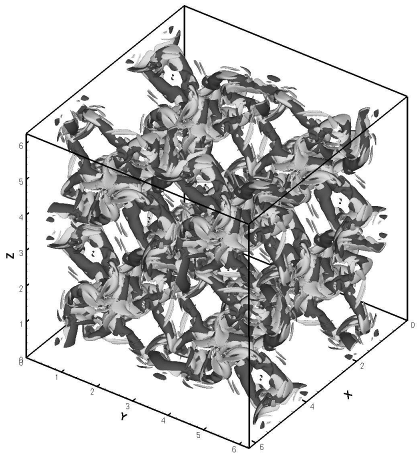

FIG. 2: Tubes (dark shades) and sheets (light shades) for Taylor-Green flow at $t = 9$ ($Re = 1000$).

range displays a collapse in Kolmogorov units; iv) in the third stage, enstrophy attains a maximum and starts to decrease. Corresponding to this time, a $k^{-5/3}$ is observed for a relatively short time interval.

In a companion poster presentation we analyze the structural properties of the flow fields in terms of vortex tubes and vortex sheets. The main finding is that the evolution of the flow field involves transition from an early stage dominated by vortex sheets, to a later stage when vorticity dissipation approximately balances production due to stretching, and vortex tubes appear. In the fully developed turbulence stage the number of vortex sheets and tubes becomes approximately equal. A typical results is shown in Fig. 2 for the Taylor-Green flow near the time of peak enstrophy.

Our ultimate objective is to look for a relation between the statistics of the vorticity field and the dynamics of elemental objects.