INVESTIGATION OF VORTEX RING WITH FINITE–AMPLITUDE KELVIN WAVES USING 3D VIC METHOD

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Summary In the paper, there will be presented a preliminary results of dynamics of vortex ring with a finite–amplitude Kelvin waves. I study an influence of the frequency of the Kelvin waves as well as the Kelvin wave amplitude on the translational velocity of inviscid vortex ring. Calculations are done for classical vortex rings with a finite thickness of a core and uniform distribution of vorticity inside it. The translational velocities of perturbed rings are compared with its unperturbed counterpart and with theoretical formula. A dynamics of rings is modeled using a vorticity particle method for inviscid flow.

VORTEX RING WITH FINITE–AMPLITUDE KELVIN WAVES

The vortex rings are the simplest and, on the other hand, ones of the most important 3D vortex structures that occur in fluid mechanics [2, 5, 6]. Despite of its simple geometry, the dynamics of vortex rings gives an interesting and good examples of non-linear interactions of the regions with concentrated vorticity. One of the most fascinating and still investigated phenomenon relates to an evolution of Kelvin waves (sinusoidal distortions) which appear on a circumference of a ring [5]. Last numerical results reported by Kiknadze and Mamaladze [3] and Barenghi et al. [1] showed that a translational velocity of perturbed vortex ring in superfluid depends on a frequency (N – wave number) and amplitude A of the Kelvin waves. Their numerical simulations refer to the case of inviscid vortex ring with infinitesimal thickness of a core which is realistic for superfluid conditions.

It was interesting to verify if the same behavior could be observed in classical inviscid and viscous fluid. Preliminary results presented in the paper refer to a classical inviscid vortex ring with finite radius of inner core.

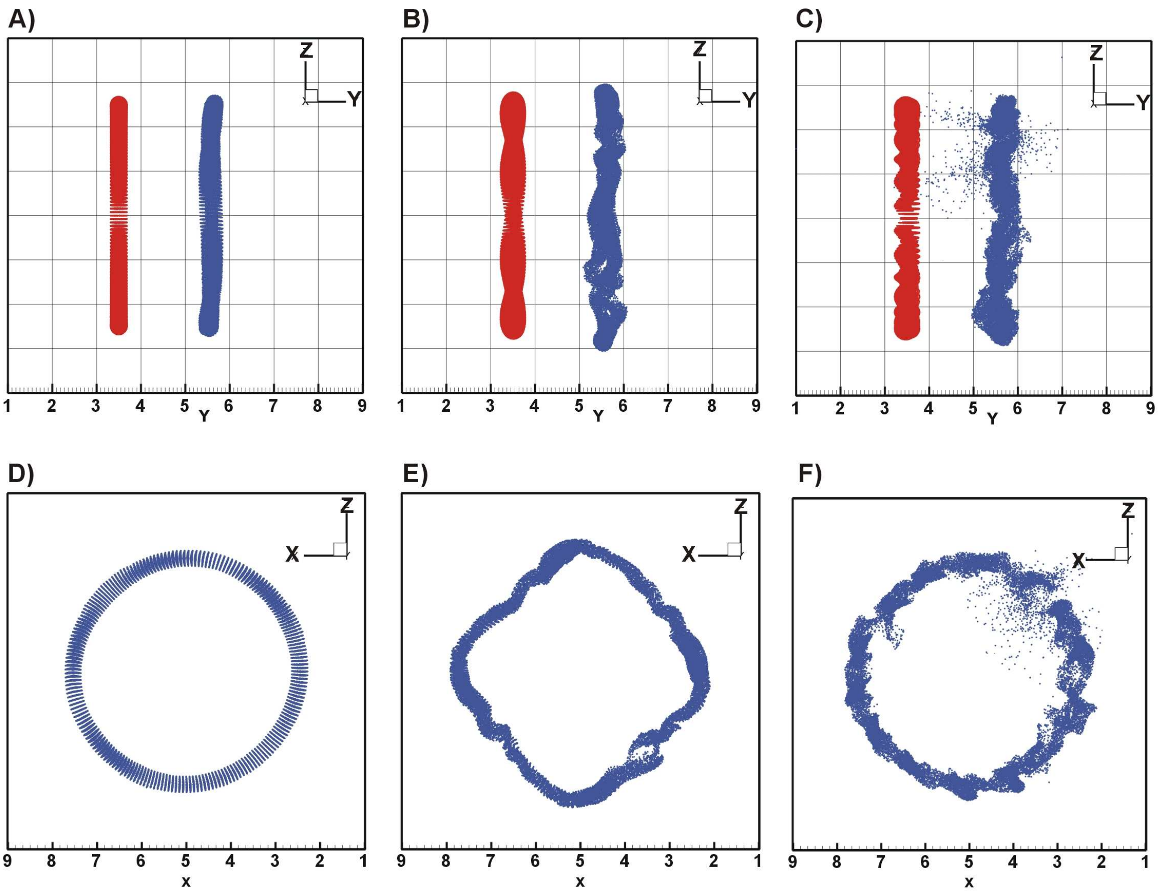

Figure 1. *A*) Side view of initial and final ($t_f = 10$) positions of unperturbed vortex ring (dark red and light blue colors respectively); *B*) Side view of initial and final ($t_f = 10$) positions of perturbed vortex ring (A/R=0.04, N=4); *C*) Side view of initial and final ($t_f = 10$) positions of perturbed vortex ring (A/R=0.04, N=14); *D*) Front view of final ($t_f = 10$) position of unperturbed vortex ring; *E*) Front view of final ($t_f = 10$) position of perturbed vortex ring (A/R=0.04, N=4); *F*) Front view of final ($t_f = 10$) position of perturbed vortex ring (A/R=0.04, N=14)

First the unperturbed vortex rings were modeled in order to compare its translational velocities with theoretical values [6] (ε, R – inner and outer radius of a ring respectively, Γ – circulation):

$$U_T = \frac{\Gamma}{4\pi R}\left[\ln\left(\frac{8R}{\varepsilon}\right) - \frac{1}{4}\right]$$

obtaining very good agreement for wide range of a different values of outer and inner radii of the ring as well as circulations Γ. Exemplary results presented in Fig. 1A) were done for $R = 2.5$, $\varepsilon = 0.2$ and $\Gamma = 1.5$. The vortex rings in Fig. 1 were approximated by a set of 24,000 vorticity particles. Next the rings perturbed by finite–amplitude Kelvin waves were modeled. Preliminary results were shown in Fig. 1B,E) and Fig. 1C,F). During the evolution, initial small distortions increased changing the shape of the rings as it is presented in Fig. 2 using iso-surfaces of vorticity.

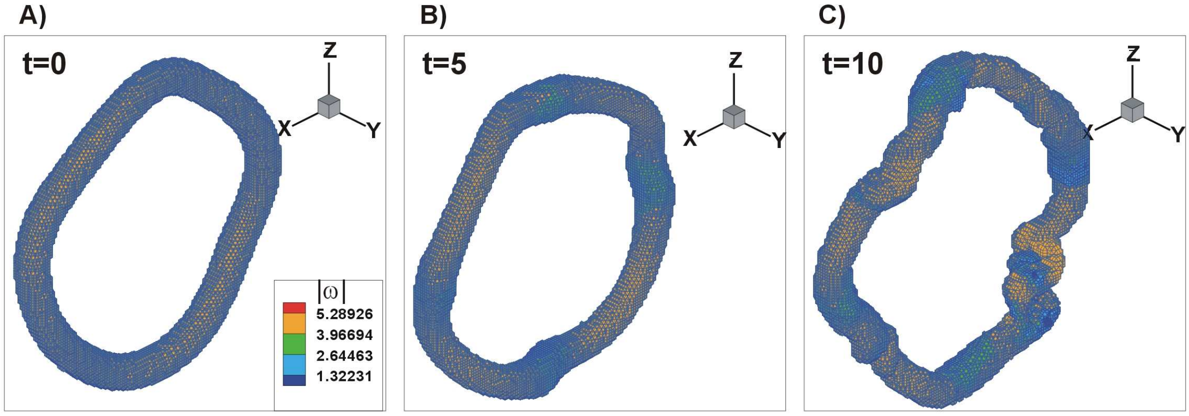

Figure 2. *A*) View of initial iso-surfaces of vorticity for perturbed vortex ring (A/R=0.04, N=4); *B*) View of iso-surfaces of vorticity for perturbed vortex ring at t=5; *C*) View of iso-surfaces of vorticity for perturbed vortex ring at t_f=10

In spite of the complicated dynamics of these structures, it seems that its translational velocities, for given parameters of ε, R, Γ, were the same regardless of perturbed or unperturbed rings. It was interesting to verify the behavior of the classical rings in a limit of its „infinitesimal" inner radii but the thickness of vortex core was limited by a grid spacing used in the calculations. Further calculations will focus on the dynamics of viscous vortex rings and the influence of viscous effects on translational velocities of perturbed structures.

VORTICITY PARTICLE–IN–CELL METHOD

For the study of the inviscid vortex ring dynamics, a vorticity particle method was used that in 2D version is known as a vortex–in–cell method [2]. One can distinguish two different types of vortex methods: the direct method based on the Biot–Savart law where the velocity of each vortex element is calculated by summing up the contribution of the all particles in the domain, and the vortex–in–cell method where the velocity is obtained on the grid nodes by solving Poisson equations for vector potential [2]. Then the vector potential is differentiated numerically by the finite difference method in order to obtain the velocity on the grid nodes and interpolate its value to the particle position. Despite the fact that vorticity is divergence free, the 3D VIC method introduces to the computation a vector particles that carries the „mass" of the vorticity [2, 4, 7]. Due to that fact during computation the divergence of the vorticity, velocity and vector potential fields were monitored. In the case of viscous flow the viscous splitting algorithm is used: at first the Euler equations are solved by VIC method and next viscous effect is taken into account by particle strength exchange method. I also monitored invariants of the motion like kinetic energy, helicity and enstrophy.

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