Viscous ring modes in vortices with jet

Stéphane LE DIZÈS[1] & David FABRE[2]
[1]IRPHE, CNRS, 49, rue F. Joliot-Curie, F-13013 Marseille, France,
[2]IMFT, CNRS, allée du Prof. Soula, F-31400 Toulouse, France.

February 29, 2008

Vortices with or without jets are present in most fluid dynamic applications and have been the subject of fundamental research for more than a century. Very recently, it was shown that viscosity can be a serious destabilising factor [1]. Unstable viscous modes localized in the centre of the vortex were shown to be generic of any vortex with jet [2, 3, 4].

In this work, we analyse a new type of viscous unstable modes which are concentrated in a ring around the vortex. A large Reynolds-number asymptotic analysis is performed for an arbitrary axisymmetric vortex with axial flow. We show that for any azimuthal wavenumber m and axial wavenumber k as soon as there exists a critical radial location $r_c > 0$ satisfying $m\Omega_c' + kW_c' = 0$ where Ω_c' and W_c' are the radial derivative at r_c of the angular and axial velocity of the vortex, there exist two or four families of viscous ring modes localized in the neighborhood of r_c. The spatial structure of these modes is as illustrated in figure 1. In each

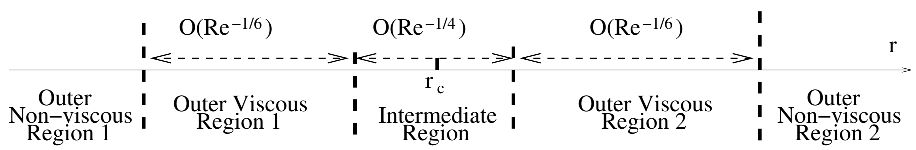

Figure 1: Asymptotic structure of viscous ring modes

region, the eigenmodes have a specific approximation. Using a matched asymptotic approach, we show that the frequency of the viscous ring modes expands as

$$\omega^{(n)\pm} \sim \omega_0 + Re^{-1/3}\omega_1 + Re^{-1/2}\omega_2^{(n)\pm} \tag{1}$$

where

$$\begin{aligned} &\omega_0 = m\Omega_c + kW_c \\ &\omega_1 = 3i(H_c/4)^{1/3}\ ; \ \text{ with } H_c = 2\Omega_c k(k(2\Omega_c + r_c\Omega_c') - mW_c'/r_c) \\ &\omega_2^{(n)\pm} = (n + \tfrac{1}{2} \pm \tfrac{\delta}{n!})\sqrt{-6iK_c}\ ; \ \text{ with } K_c = m\Omega_c'' + kW_c'',\ \delta \ll 1 \end{aligned} \tag{2}$$

Viscous ring modes resemble the viscous centre modes described using the same approach in [3]. In particular, we show that unstable ring modes are mainly localized in the outer viscous regions and have the same leading order approximation as unstable viscous centre modes in these regions.

The theoretical predictions are compared to numerical results for different vortex models, including the q-vortex ($\Omega(r) = q(1 - \exp(-r^2))/r^2$; $W(r) = \exp(-r^2)$). The numerical results are obtained with the spectral collocation code used in [3]. For all modes, a good agreement is demonstrated for both the frequency and the spatial structure. An example of such a comparison is provided in figure 2 for the q-vortex. For this vortex, viscous ring modes are found to be unstable in a smaller domain than viscous centre mode instability domain and possess smaller growth rates.

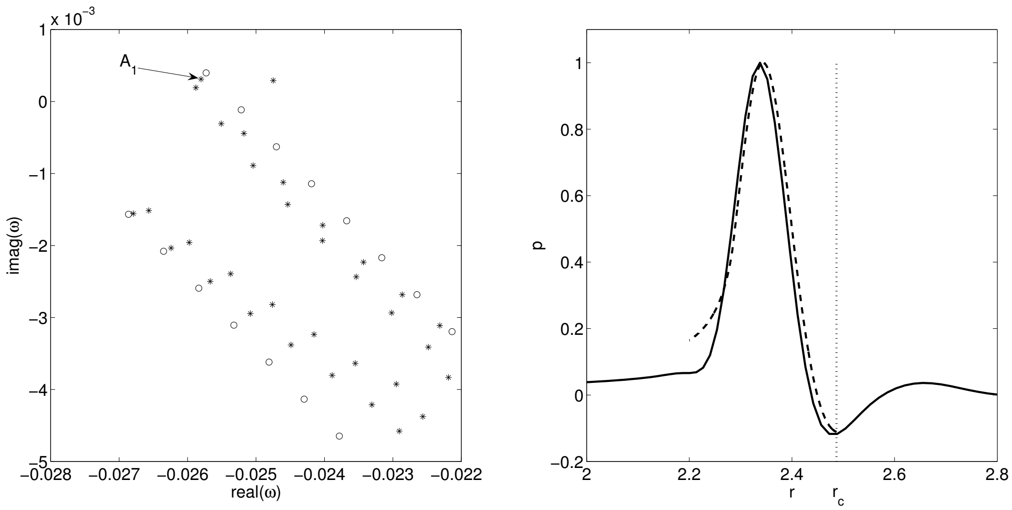

Figure 2: Typical results for the q-vortex model. $Re = 10^6$, $m = -1$, $k = 2.5$, $q = 0.2$. (a) Frequency versus growth rate of the viscous ring modes. Stars: numerical results, circles: asymptotic results [formulae (1) and (2) with $\delta = 0$]. (b) Real part of the pressure amplitude of the mode A_1 indicated in (a). Solid line : numerical results, Dashed line: asymptotic approximation in the Outer Viscous Region 1.

For other vortices, viscous ring modes can be more unstable than viscous centre modes. However, viscous ring modes are always in competition with inviscid modes. We show that for sufficiently large Reynolds numbers, a necessary and sufficient condition of instability of viscous ring modes is that there exists a location r_c where

$$\Omega_c \Omega_c' [r_c \Omega_c' (2\Omega_c + r_c \Omega_c') + (W_c')^2] < 0 \ . \tag{3}$$

This condition is exactly the sufficient condition of inviscid instability obtained by Leibovich & Stewartson [5]. We analyse the interplay between both instabilities, especially in the limit of large m, for which both types of mode have the same leading order frequency and are localized near the same critical radial location r_c.

References

[1] D. Fabre and L. Jacquin, "Viscous instabilities in trailing vortices at large swirl numbers," J. Fluid Mech. **500,** 239–262 (2004).

[2] K. Stewartson, T. W. Ng, and S. N. Brown, "Viscous centre modes in the stability of swirling Poiseuille flow," Phil. Trans. R. Soc. Lond. A **324,** 473–512 (1988).

[3] S. Le Dizès and D. Fabre, "Large-Reynolds-number asymptotic analysis of viscous centre modes in vortices," J. Fluid Mech. **585,** 153–180 (2007).

[4] D. Fabre and S. Le Dizès, "Viscous and inviscid centre modes in the linear stability of vortices: the vicinity of the neutral curves," J. Fluid Mech. (2008), in press.

[5] S. Leibovich and K. Stewartson, "A sufficient condition for the instability of columnar vortices," J. Fluid Mech. **126,** 335–356 (1983).