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Lagrangian Panel Method for Vortex Sheet Motion in 3D Flow

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Caflisch and Kaneda derived the evolution equation for vortex sheet motion in 3D flow [1, 2]. The sheet is a parametrized surface, $\mathbf{x}(\alpha, \beta, t)$, with Lagrangian parameters α, β and time t, and the equation has the form

$$\partial_t \mathbf{x} = \int_0^{2\pi} \int_0^1 \mathbf{K}_\delta(\mathbf{x}, \tilde{\mathbf{x}}) \times d\tilde{\mathbf{\Gamma}}, \tag{1}$$

where $\mathbf{\Gamma}(\alpha, \beta)$ is the vector-valued circulation. We use a regularized Biot-Savart kernel,

$$\mathbf{K}_\delta(\mathbf{x}, \mathbf{y}) = -\frac{\mathbf{x} - \mathbf{y}}{4\pi(|\mathbf{x} - \mathbf{y}|^2 + \delta^2)^{3/2}}, \tag{2}$$

where δ is a smoothing parameter. The poster will describe a new Lagrangian panel method for tracking the sheet surface [3]. The sheet is represented by a set of quadrilateral panels having a quadtree structure. The panels have active particles that carry circulation and passive particles for adaptive refinement. The velocity is evaluated by a treecode [4]. The method was applied to compute the azimuthal instability of a vortex ring [3]. Figure 1 plots the results at $t = 0, 4, 8$. Initially the sheet is a circular disk with an azimuthal perturbation. The edge of the sheet rolls up into a spiral, effectively forming a vortex ring. To help visualize the core dynamics, a set of material lines was tracked in the flow induced by the vortex sheet; these are the colored lines in rows 3 and 4 of Figure 1. The motion of the green lines indicates local axial flow, a feature seen in experiments by Naitoh et al. [5]. At late times a sequence of dipoles is being ejected from the main ring structure, another feature possibly seen in experiments by Dazin et al. [6].

References

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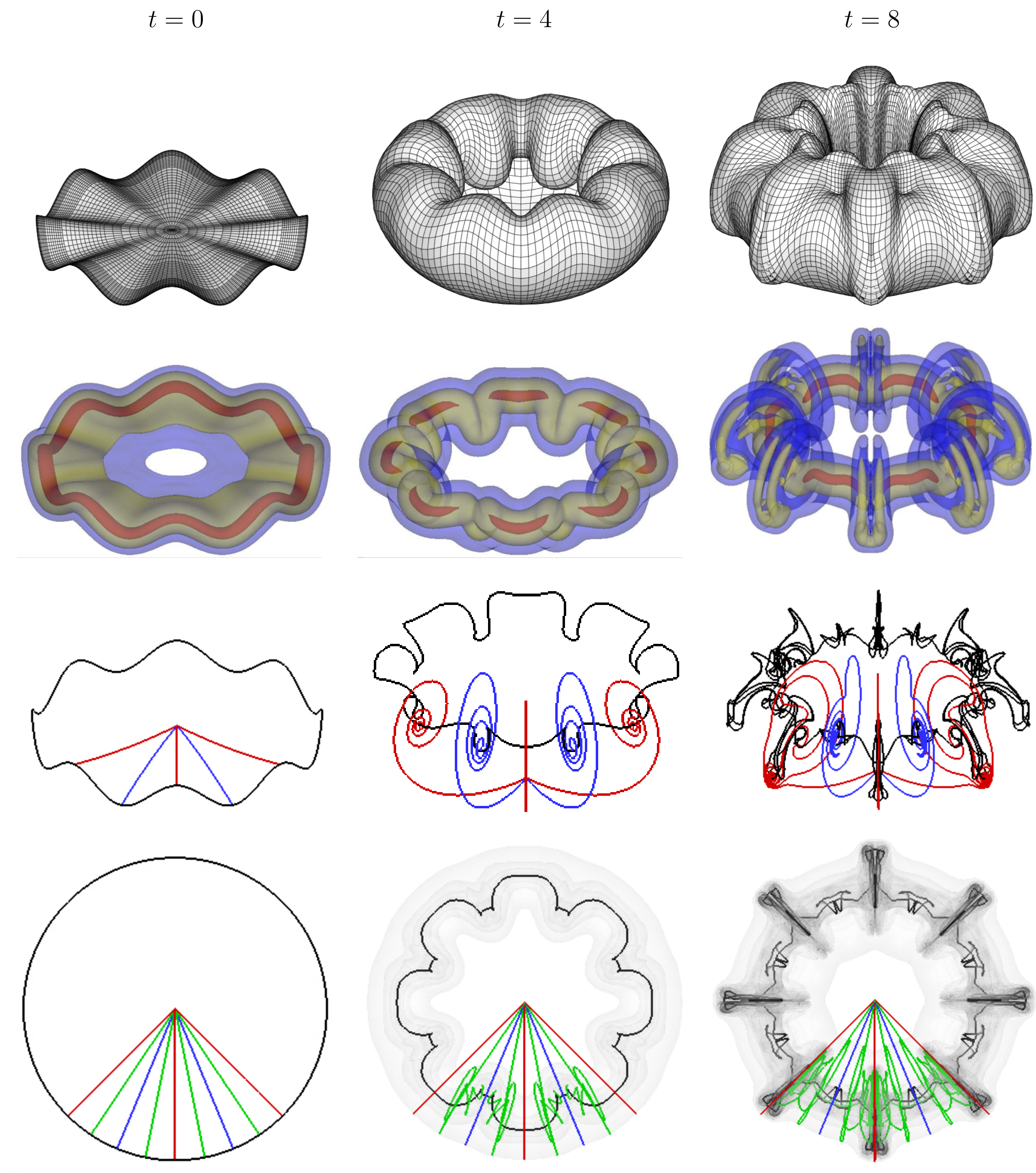

Figure 1: Azimuthal instability of a vortex ring computed by a vortex sheet panel method [3], $t = 0, 4, 8$ (left to right). row 1: computational panels; row 2: vorticity isosurfaces; row 3: a black material line tracks the vortex core, blue and red material lines roll up around the core; row 4: (top view) blue and red material lines stay in their initial plane while green material lines move out of their initial plane, an indication of local axial flow, a feature seen in experiments by Naitoh et al. [5]. At late times a sequence of dipoles is being ejected from the main ring structure, another feature possibly seen in experiments by Dazin et al. [6].